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APPLICATION OF THE HARTMANN TEST TO THE MEASUREMENT OF OBLIQUE ABERRATIONS

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ABSTRACT. By performing a Hartmann test on an oblique pencil of light through a lens it is possible to determine the coma and astigmatism with great simplicity and accuracy. The theory of the test has been recently investigated by Prof. A. E. Conrady, and an outline of his proof of the formulae involved is given, together with the complete numerical results of such a test applied to a small inclined telescope objective. The measures of coma and astigmatism obtained for this lens have a probable error of the order of 1 per cent.

INTRODUCTION

THE Hartmann test, as is well known, is a method of experimental ray-tracing, the "rays" being actually very thin pencils of light passing through small holes in a metal diaphragm placed in the plane of the entrance or exit pupil of the lens.

In order to determine the exact positions of the intersections of the various "rays" with one another, two photographs are taken, one in a plane lying a short distance in front of the image, and the other in a plane lying about the same distance beyond the image. These photographs when developed are seen to consist of patterns of dots having the same general shape as the arrangement of holes in the diaphragm at the lens, the inside photograph being of course the same way up, and the outside photograph inverted.

Usually the Hartmann test is applied to an axial pencil of rays through the lens, to measure the spherical aberration; but it may also be very conveniently employed to determine the distribution of light at an image which lies considerably off the axis. In this case the most convenient form of the diaphragm at the lens is one having a central hole with one or more concentric rings of holes to mark zones on

the lens. The holes should be equally spaced round the rings, their number being divisible by four in order that holes may fall on the tangential and sagittal axes, called respectively the y and x axes. The sagittal or x-axis coincides with the mechanical axis of rotation about which the lens has been tilted.

The D-Pairs

Throughout this paper a pair of holes in the diaphragm at the same height above the sagittal axis and at equal distances on either side of the tangential axis will be called a "D-pair" (Fig. 1). The purpose of this test is to locate the intersection points of rays from the two component holes of all such pairs; or supposing the two rays of a pair intersect at a point D, then it is required to determine experimentally the *longitudinal aberration* LA_D of this point from the Petzval surface, and also its *transverse aberration* TA_D from the central ray of the pencil. The central ray is taken as being the thin beam of light passing through the hole in the centre of the metal diaphragm, and is often called the principal ray. A knowledge of the LA and TA of the various D-pair foci enables us to calculate the aberration coefficients of the lens.

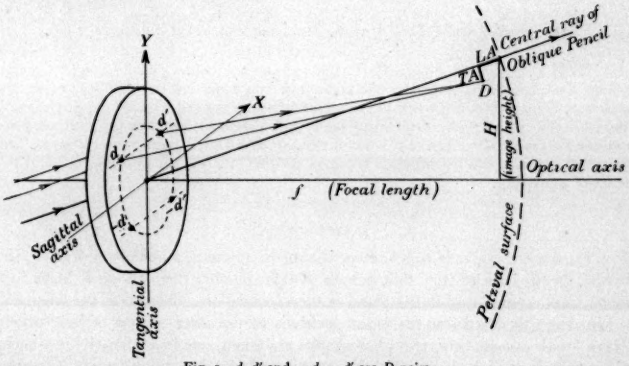


Fig. 1. d, d' and $-d, -d'$ are D-pairs

Theory of the Test

Prof. Conrady has shown that for a telescope objective at small obliquity the LA_D and TA_D of a D-pair are related to the primary aberration coefficients a_1, a_2, and a_3 by the following equations:

$$\left.\begin{aligned} LA_D &= a_1 S^2 f^2 + 2a_2 f Y H + a_3 H^2 \\ TA_D &= a_2 S^2 H + 2\frac{a_3}{f} Y H^2 \end{aligned}\right\} \quad \ldots\ldots(1),$$

where S = semi-aperture of the zone of the lens containing the D-pair,
f = focal length of the lens (assuming object at infinity),
Y = height of the D-pair above the sagittal axis,
H = height of the image above the optical axis of the system,

and a_1, a_2, and a_3 are the aberration coefficients of spherical aberration, coma, and astigmatism respectively. These are the same coefficients that appear in Prof. Conrady's equation for the aberrations in terms of differences of optical path*.

The rigorous proof of these equations is rather long and is as yet unpublished. Prof. Conrady has, however, been good enough to permit me to give an outline of his proof for the benefit of those who do not care to take such comprehensive formulae merely on trust.

The proof depends on the fact that the longitudinal spherical aberration arising at a surface is proportional to the square of the aperture, hence if we represent longitudinal aberration by LA and semi-aperture by S, we have

$$LA' = aS^2 \qquad \text{.....(i)},$$

where a is a constant. (Throughout this proof, a dash after a symbol implies that it refers to the image side of a surface.) Prof. Conrady† has shown that the primary spherical aberration arising at a surface is

$$LA' = LA\frac{N}{N'}\left(\frac{u}{u'}\right)^2 + \frac{1}{2}\frac{l'i''}{u'}(i-u')(i-i'),$$

where
N = refractive index
l = intersection length
u = convergence angle } of a paraxial ray.
i = incidence angle

Then, for an object-point free from aberration, since $S = l'u'$,

$$LA' = aS^2 = al'^2u'^2 = \frac{1}{2}\frac{l'}{u'}i'(i-u')(i-i').$$

$$\therefore \quad a = \frac{1}{2l'}\frac{i''}{u'^3}(i-u')(i-i') \qquad \text{.....(ii)}.$$

Knowing this value of a, we can now determine at once the primary longitudinal aberration arising at any surface through which we have traced one paraxial ray.

To extend this result to an oblique pencil, we must consider an auxiliary axis drawn from the oblique object-point (Fig. 2) through the centre of curvature of the surface. Let A be the pole of our surface of radius r and centre C, and suppose that an axial object-point B produces a paraxial image at B'. We will now consider an extra-axial object-point B_0, situated at a distance H from B, lying on the auxiliary axis B_0C. A paraxial ray from B_0 (shown dotted) will meet this axis again at some point B'_0, which will lie nearer the surface than B' because the object B_0 is farther from it than B. The images B'_0 and B' lie on the Petzval surface. We will also suppose that a diaphragm is placed at E to limit the entering pencil of light, and

* *Trans. Opt. Soc.* 27 (1925–26) 94.

† This is the first approximation of the strict formula for spherical aberration given in *Dictionary of Applied Physics*, vol. 4, p. 210, equation (9*).

that the *principal ray* from B_0 through the centre of the diaphragm meets the surface at P, and forms an image on the auxiliary axis at B'_p. B'_p will not coincide with B'_0 because of spherical aberration, and to calculate the magnitude of this separation, we apply the rule of equation (i), noting that the incidence-height of the principal ray is PA_0 approximately. But, to ensure that the aberrations will be strictly primary, it is necessary to assume that the aperture of the lens and the inclination of the pencil are both small. All quantities are considered positive or negative in the usual analytical sense about A as origin.

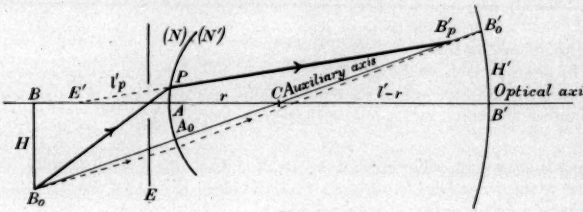


Fig. 2

Then $PA_0 = PA + AA_0$

$$= H' \frac{l'_p}{l'_p - l'} + H' \frac{r}{l' - r} = H' \left(\frac{l'_p}{l'_p - l'} + \frac{r}{l' - r} \right) = qH' \quad \text{.....(iii).}$$

q is called the eccentricity-constant for the surface, and is easily calculated from the tracing of the principal ray and a paraxial ray. Hence, by equation (i), the longitudinal aberration of the principal ray measured back from the Petzval surface is

$$LA'_p = B'_p \cdot B'_0 = a\,(A_0P)^2 = aq^2H'^2 \quad \text{.....(iv).}$$

In the same manner we can now determine the longitudinal aberration of any other ray from B_0 through the entrance-pupil E, defined by the S and Y of the point D or D' at which it strikes the surface (Fig. 3).

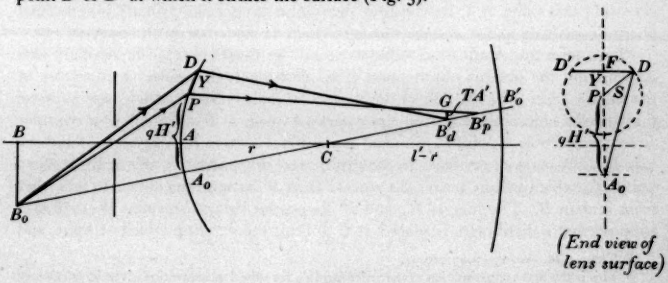


Fig. 3

Then, by our rule, the longitudinal aberration of this ray will be

$$\begin{aligned} LA'_D &= a\,(A_0D)^2 \\ &= a\,[(A_0F)^2 + (FD)^2] \text{ by Pythagoras} \\ &= a\,[(qH' + Y)^2 + (S^2 - Y^2)]. \end{aligned}$$

$$\therefore\ LA'_D = aq^2H'^2 + 2aYqH' + aS^2 \qquad \text{.....(v).}$$

But, since the auxiliary axis changes from surface to surface through a lens system, it is better to refer the image of our D-ray to the principal ray, and to do this it is necessary to calculate the transverse aberration TA'_D, which is the distance of B'_D below the principal ray.

Then $$TA'_D = \frac{qH'}{l'}\,(B'_DB'_p) = \frac{qH'}{l'}\,(LA'_D - LA'_p)$$

$$= \frac{qH'}{l'}\,(aS^2 + 2aYqH') \text{ by equations (iv) and (v).}$$

$$\therefore\ TA'_D = \frac{aq}{l'}\,H'S^2 + 2a\frac{q}{l'}\,H'^2Y \qquad \text{.....(vi).}$$

So much for one surface. In a complete lens system, the contributions of each surface to the final LA and TA are found by applying to the LA and TA of that surface the laws of longitudinal and transverse magnification

$$\frac{N'}{N_0}\left(\frac{u'}{u_0}\right)^2 \text{ and } \frac{N'}{N_0}\left(\frac{u'}{u_0}\right) \text{ respectively,}$$

where suffix o refers to the image side of the last surface. Hence the contributions of a surface to the final image are

$$\left.\begin{aligned} &\text{towards } LA\text{:}\quad (aq^2H'^2 + 2aYqH' + aS^2)\,\frac{N'}{N_0}\left(\frac{u'}{u_0}\right)^2 \\ &\text{towards } TA\text{:}\quad \left(\frac{aq}{l'}\,H'S^2 + 2a\frac{q}{l'}\,H'^2Y\right)\frac{N'}{N_0}\left(\frac{u'}{u_0}\right) \end{aligned}\right\} \qquad \text{.....(vii).}$$

In order to sum up these contributions from all the surfaces, the H, S, and Y must be expressed in terms of H_0, S_0, and Y_0 respectively. By the Lagrange invariant $HNu = H'N'u'$ we have

$$H' = H_0\frac{N_0u_0}{N'u'}.$$

Also, since $S = l'u'$ and $S_0 = l_0u_0$,

$$S = S_0\frac{l'u'}{l_0u_0}$$

and also

$$Y = Y_0\frac{S}{S_0} = Y_0\frac{l'u'}{l_0u_0}.$$

On substituting these values in equation (vii) and adding up the contributions of all the surfaces, we get

$$\left.\begin{aligned} \text{Final } LA_D &= \frac{l_0^2S_0^2}{N_0}\,\Sigma\frac{aN'l'^2u'^4}{l_0^4u_0^4} + 2Y_0H_0l_0\,\Sigma\frac{aql'u'^2}{l_0^2u_0^2} + H_0^2N_0\,\Sigma\frac{aq}{N'} \\ \text{Final } TA_D &= S_0^2H_0\,\Sigma\frac{aql'u'^2}{l_0^2u_0^2} + 2Y_0H_0^2\frac{N_0}{l_0}\,\Sigma\frac{aq}{N'} \end{aligned}\right\}$$

or, writing a_1, a_2, and a_3 for the three Σ-sums, dropping the suffix o, writing $N_0 = 1$ since the final image is in air, and putting $l' = f$, the focal length of the telescope objective,

$$\left.\begin{aligned} LA_D &= a_1 S^2 f^2 + 2a_2 fYH + a_3 H^2 \\ TA_D &= a_2 S^2 H + 2\frac{a_3}{f} YH^2 \end{aligned}\right\}$$

which are the formulae given above as equation (1).

The special case of a D-pair lying actually in the sagittal axis of the lens may be regarded as simply a D-pair with $Y = 0$. If $Y = 0$ is substituted in (1), we get for a *Sagittal Pair*:

$$\left.\begin{aligned} LA_S &= a_1 S^2 f^2 + a_3 H \\ TA_S &= a_2 S^2 H \end{aligned}\right\} \qquad \ldots\ldots(2).$$

This quantity TA_S is called the "sagittal coma" of the lens, and, as is evident from the equation, a knowledge of its value gives us an immediate solution for the coma coefficient a_2.

Determination of the Longitudinal and Transverse Aberrations from the Hartmann Photographs

The oblique Hartmann test to be described is a method for determining experimentally the values of LA_D and TA_D for a series of D-pairs on a lens. By means of the formulae given above it is then possible to calculate a_2 and a_3, which are measures of the absolute amounts of coma and astigmatism present in the lens. It is not usually convenient to determine the spherical aberration from an oblique test, so that if a measure of spherical aberration is desired, this should be determined by an independent axial Hartmann test, using a diaphragm with a number of pairs of holes at different distances from the centre.

We will now show what measurements must be made on the two Hartmann photographs to enable us to calculate the longitudinal and transverse aberrations of the D-pairs. Consider the case of a metal diaphragm containing only a central hole and one D-pair. Then the paths of the rays in the neighbourhood of the image, from these three holes, will be as shown in Fig. 4. The two photographs will each consist of three dots, shown in the diagram by the points P_1, Q_1, O_1 on the inner plate, and by P_2, Q_2, O_2 on the outer plate. The point D, the intersection of the "rays" P_1P_2 and Q_1Q_2, is evidently the focus of the D-pair; and, as we have seen, it is required to determine the values of the transverse aberration T, or the distance of D below the central ray O_1O_2, and also the longitudinal aberration of D from the Petzval surface. But, since we cannot locate this Petzval surface experimentally, we can only determine a, the distance of D from the outer Hartmann plate, where $a = LA_D +$ (const.).

We will call the separation between the images of the D-pair on each plate l, and the height of those images above the central image h (Fig. 4). Then if we regard

the system in plan, the plates and rays would appear as in Fig. 5 *a*, where evidently by similar triangles,

$$a = \frac{l_2}{l_1 + l_2} d \qquad \text{.....(3).}$$

If we view the system in side elevation (Fig. 5 *b*), the rays P_1P_2 and Q_1Q_2 appear to coincide, and by similar triangles we see that

$$T = (h_2 - h_1)\frac{a}{d} - h_2 \qquad \text{.....(4),}$$

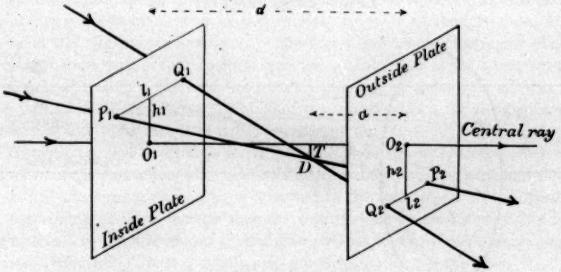
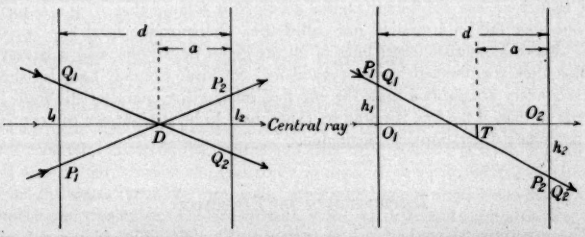


Fig. 4. Path of rays in the neighbourhood of the image of a *D*-pair



(*a*) Plan view Fig. 5 (*b*) Side elevation

remembering by the sign-convention that h_1 in Fig. 4 is shown positive whereas h_2 is negative. The transverse aberration T must, however, be regarded as positive if D falls below the central ray O_1O_2. This is because D falls below O_1O_2 in an uncorrected lens, and it is customary to regard such a lens as having positive aberrations. Hence the only measurements that need be made on the plates are the separations l and heights h of the dot-images of the various D-pairs.

The Characteristic Focal Line

Prof. Conrady has pointed out a very interesting deduction from the theory given above, that the foci of all possible D-pairs lying in a zone of the lens fall along an inclined straight line in the plane of the optical axis. For, if we consider a straight line joining D, the focus of a D-pair, to the sagittal focus S of the same zone (Fig. 6), the angle θ between this line and the central ray is given by

$$\tan\theta = \frac{TA_D - TA_S}{LA_D - LA_S} = \frac{2\frac{a_3}{f}YH^2}{2a_1fYH} = \frac{a_3}{a_2}\cdot\frac{H}{f^2}.$$

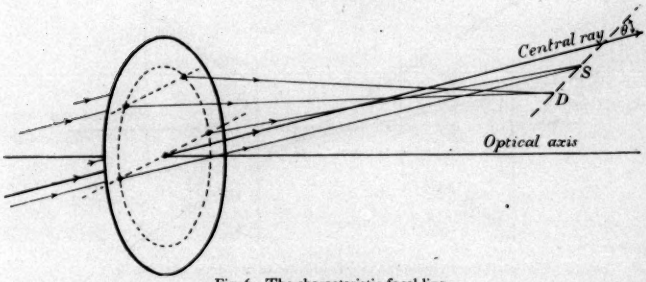


Fig. 6. The characteristic focal line

We see therefore that this angle is constant for all the D-pairs of all the zones, and since each zone has a different sagittal focus S, it follows that all the D-foci of any one zone fall on a straight line called the "characteristic focal line" of that zone. The characteristic focal lines of all the zones lie parallel, and it is easy to see that they are bisected by the sagittal foci S. Prof. Conrady has also shown that secondary aberrations have the effect of making the characteristic focal line curved; so that, if the line is found by experiment to be straight, we can conclude that secondary aberrations are negligible.

Description of the Experiment

The experimental method is best exemplified by describing in some detail a test performed on a good $1\frac{1}{2}$" telescope objective of about 12" focal length, being one of Cooke's pathological set. Since it was desired to study an oblique pencil, some device was necessary to enable the lens to be tilted by a known amount up to about 5°. The apparatus for this consisted of a screwed lens cell in a circular plate, which rested on two small balls by a geometric hole and slot arrangement so as to allow one degree of freedom, namely a rotation about an axis joining the centres of the balls. The tilting movement was caused by a micrometer screw

pressing on one side of the disc, while a barrel-spring pressing on the other side maintained everything in its correct position.

The source of light was an artificial star made by focusing a mercury vapour lamp on to a small hole in a piece of thin brass. To eliminate all chromatic effects in the lens, a 60° prism was used to spread out the mercury light into a spectrum on the brass plate, only the violet line being allowed to pass through the pinhole. This star was erected at one end of a long passage from which all other light was carefully excluded to prevent fogging of the plate.

The test lens was mounted in its tilting holder at the front of a telescope fitted with a sliding draw-tube. The photographic plate, 2 cm. square, was held in a little plate-holder made to fit in place of the ordinary eyepiece of the telescope. Having discovered the approximate position of the focus, the inside Hartmann plate was taken about 8 cm. within that position, and the outside plate about 8 cm. beyond it. The actual separation d between the plates was carefully measured. It was found convenient to mark the draw-tube so that the two positions for the plates would be known at any time without further measurement.

The telescope was erected as far as possible from the star, at the other end of the dark passage, and adjusted to give an image of the star in the centre of the field. The drilled diaphragm having been attached to the back surface of the lens with wax, care being taken to ensure that the sagittal axis coincided with the axis of tilt, the lens was tilted by about 5°, and the two photographs were taken on pieces of Ilford "Monarch" backed plate, with an exposure of 2–3 minutes. The actual tilt of the objective was subsequently measured on a spectrometer by autocollimation from a piece of plane mirror attached to the lens cell.

Before drilling the diaphragm, experiments were performed to determine the best size of hole, and the least separation of the holes that would give no diffraction bands between the dots on the plates. Eventually it was found that the best size of hole was that made with drill No. 54 (1·4 mm. diameter) and the least safe separation was 3 mm. This agreed with Hartmann's original estimate* that the diameter of the holes should be about $\frac{1}{200}$ of the focal length. The final diaphragm was made up with a central hole, a zone carrying 16 holes with a radius of 1·3 cm., and a smaller zone of 12 holes at a radius of 0·6 cm., care being taken that holes in each zone fell exactly on the sagittal and tangential axes of the lens. A guide hole was also drilled between the two zones to assist in determining the directions in which the plates should be measured. After marking out a thin brass disc with the utmost care, the holes were drilled with a No. 54 drill, and countersunk to make the edges as thin as possible. The disc was then polished and bronzed to minimise stray reflections during exposure. In addition to measurements on the photographs, the Y's and semi-apertures of the D-pairs on the two zones were required, which were obtained from the metal diaphragm with a measuring microscope.

* *Z. f. Instrk.* 24 (1904) 1.

Measurement of the Photographs

To save confusion, the holes in the diaphragm were lettered, corresponding letters being marked against the dots on the photographs (Fig. 7). The photographic plate to be measured was mounted on the table of a Cambridge and Paul micrometer measuring microscope, with the eyepiece cross-wire parallel to the *D*-pairs *aa'*, etc., and settings were made successively on each pair and on the central hole. Subtraction of the central reading from each of the others then gave the value of *h* for each *D*-pair. To obtain the values of *l*, the plate was turned round through a right-angle until the cross-wire lay parallel to the tangential axis, and settings were made on each dot. Subtraction of the reading for a dot on one side from that of the corresponding dot on the other side gave the value of *l* for the pair. This provided all the information necessary for the application of the above formulae. In each case at least five or six settings were made and the average taken.

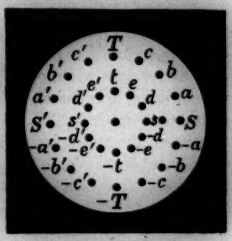


Fig. 7. Notation of diaphragm holes (viewed from object side of lens)

The ultimately found values of *h* and *l* are given in the Table, together with the corresponding measures of *Y* and *S* from the metal diaphragm, and also the results of calculation of the values of *TA* and *a* for the *D*-pairs by equations (3) and (4).

Separation of Hartmann plates, d = 16·75 cm.

D-pair	Diaphragm		Inner plate		Outer plate		Results	
	S	*Y*	h_1	l_1	h_2	l_2	*a*	*TA*
	cm.	cm.	mm.	mm.	mm.	mm.	cm.	mm.
cc'		1·1984	2·845	2·490	−4·000	3·170	9·3812	·1664
bb'		·9173	2·173	4·590	−3·040	5·738	9·3059	·1438
aa'		·4964	1·138	6·122	−1·634	7·421	9·1782	·1151
SS'	1·2970	0	− ·073	6·666	− ·081	7·806	9·0348	·0767
−*aa'*	(outer zone)	− ·4964	−1·300	6·238	1·389	7·055	8·8900	·0382
−*bb'*		− ·9173	−2·424	4·790	2·638	5·262	8·7684	·0118
−*cc'*		−1·1984	−3·141	2·618	3·414	2·826	8·6950	−·0113
ee'		·5258	1·264	1·522	−1·658	1·859	9·2100	·0514
dd'	·6071	·3035	·722	2·702	− ·946	3·247	9·1424	·0356
ss'	(inner zone)	0	− ·046	3·136	·032	3·688	9·0525	·0102
−*dd'*		− ·3035	− ·785	2·698	·920	3·109	8·9679	−·0072
−*ee'*		− ·5258	−1·346	1·569	1·584	1·778	8·8980	−·0275

The most interesting way of expressing these results is to draw a graph connecting *a* with *TA* (Fig. 8), which is in fact a side view of the image-points of the *D*-pairs. It can be seen at once that for any zone these image points lie on a straight

line bisected by the sagittal focus S. The straightness of these characteristic focal lines confirms the absence of appreciable secondary aberrations.

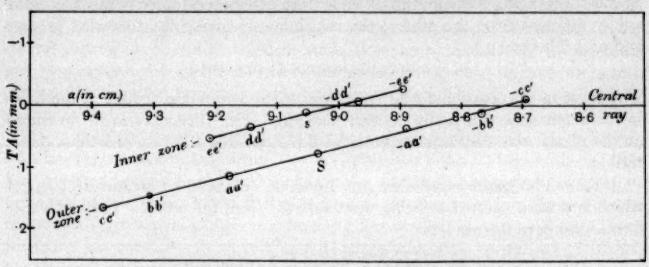


Fig. 8. The characteristic focal lines determined for the experimental lens

Calculation of the Aberration Coefficients

The theoretical equations (1) and (2) are not quite suitable for direct calculation of a_2 and a_3, so we will now deduce three special equations, suitable for the direct calculation of a_2 or a_3 by having only one of these coefficients in each. In deducing such equations it should be noted that, since we do not know LA itself but only $a = LA + (\text{const.})$, LA may only appear in the form of the difference between two independently determined longitudinal aberrations. Then we may substitute the difference between the calculated values of a for the required difference of longitudinal aberrations.

From equations (1) and (2) we obtain by subtraction

$$LA_D - LA_S = 2a_2 fYH \qquad \ldots\ldots(\alpha),$$

and

$$TA_D - TA_S = 2\frac{a_3}{f} YH^2 \qquad \ldots\ldots(\beta).$$

Also, since for a D-pair above the axis and the corresponding D-pair below the axis, all our constants are unchanged except that Y changes its sign, we have

$$TA_D = a_2 S^2 H + 2\frac{a_3}{f} YH^2$$

and

$$TA_{-D} = a_2 S^2 H - 2\frac{a_3}{f} YH^2,$$

whence by addition we obtain another useful equation:

$$TA_D + TA_{-D} = 2a_2 S^2 H \qquad \ldots\ldots(\gamma).$$

Before substituting our values of LA and TA in these equations, it was necessary to measure the focal length f and the image-height H. Actually the focal length of the lens used was found to be 30·55 cm., while the angle of inclination was 5° 3′, giving as the value of the image-height

$$H = 30{\cdot}55 \tan 5^\circ 3' = 2{\cdot}6997 \text{ cm.}$$

(*a*) *The Coma-coefficient* a_2. From the figures given in the table it was found that equation (α) gave 10 measures of a_2, and equation (γ) gave 5 measures, or 15 measures in all. Actually two of these from equation (γ) were rejected as being wildly different from the others; the remaining 13 giving the following average and probable error:

$$a_2 = \cdot 001752 \pm \cdot 000008 \ (\tfrac{1}{2} \text{ per cent.}).$$

The cause of the occasional wild errors was partly inaccurate drilling of the holes in the diaphragm, and partly the fact that some of the distances to be measured on the plates were very small, approaching the possible accuracy of a micrometer setting.

(*b*) *The Astigmatism-coefficient* a_3. Equation (β) gave 10 measures of a_3, of which two were rejected as being very different from the others. The remaining 8 measures gave the result:

$$a_3 = \cdot 01598 \pm \cdot 00019 \ (1 \cdot 1 \text{ per cent.}).$$

Conclusion

In conclusion we may say that this oblique Hartmann test is a convenient and straightforward method of measuring the oblique primary aberrations of a lens, giving an accuracy of about 1 per cent. with ordinary precautions and care. It should be mentioned, for the benefit of anyone proposing to try the test for himself, that the utmost care is required in marking out and drilling the diaphragm, since all the formulae depend on the two components of a *D*-pair being equidistant from the centre and at the same height above the sagittal axis. If these conditions are not fulfilled, it is hopeless to expect accurate results from the calculations.

This modification of the Hartmann test has been recently repeated by other students at the Optics Department of the Imperial College; it has also been applied to an oblique pencil through a rectilinear camera lens and to an axial beam through a toric lens by Miss H. G. Conrady.

I must express my best thanks to Prof. Conrady for permission to give a summary of his recent work on the primary aberrations, and also for the help he has freely given me during the course of the experimental part of the work.

DISCUSSION

Mr H. H. Emsley (communicated): Both in this paper and the previous one he read, Mr Kingslake has described work that is, I am sure, of great interest to many members of the Society. To take actual lenses and arrive at figures by tests such as the Hartmann—the principles of which can be easily grasped—and to compare the results with the usual expressions, simplified or put into such form that those without an extensive mathematical equipment can follow them, is really useful work that many are glad to see carried out. In trying to give to elementary students

a qualitative idea, and in a rough way a quantitative idea also, of these primary lens defects, I use a large lens of 11 inches diameter, of almost plano-convex form, and made of quite ordinary plate. By using a series of diaphragms with rings of holes arranged around circles of varying diameters, a "Pointolite" lamp several metres away, and a screen movable along an optical bench, the students carry out experiments enabling them to understand in a general way the interesting paths of the refracted rays and to make measurements sufficiently accurate to show the dependence of the aberrations on aperture, obliquity, position of stops, etc.

It is when the student attempts to get hold of expressions from which he can calculate these quantities, however, that his difficulties begin. There is no standard English work leading him from his elementary geometrical and physical optics to problems of this kind. It has always seemed to me that there is a distinct gap in the subject here which, looked at from any point of view, should be filled. There are many advanced works on instrument design and other specialised problems, but the ordinary type of student I have in mind cannot follow them, partly because of the gap alluded to and partly because of the variety of symbols and conventions used.

It is only in regard to this phase of the matter that I would like to make anything like criticism of Mr Kingslake's paper. I would submit that it would assist—ophthalmic students especially so, and they form a large proportion of those interested in optics—if distances were not designated by capital letters, e.g. H, Y, etc. in this paper. Why is capital N used for refractive index? With regard to the principal ray, I gathered that the term "chief" ray, suggested by S. P. Thompson, was to be used for this.

The question of nomenclature and conventions in optics has been raised many times, but I would suggest that the time is ripe for it to be seriously tackled and settled. And if, still further, a textbook could be produced to bridge the gap I have mentioned, a great step forward would have been made in the sphere of optics in this country. Such a work would be best prepared by one who is an expert with a comprehensive grasp of technical problems and at the same time is a teacher of the type of student referred to, or by collaboration between two people who share these qualifications.

Prof. L. C. Martin: I would like to draw attention to the important nature of the present paper. The new guise under which the Seidel aberrations appear, when discussed in this way by Prof. Conrady's theory, renders possible a much clearer mental conception of the various defects as expressed in ray paths, and points the way immediately to the experimental determination of the aberration constants.

The work is by no means difficult, and two of the advanced students in the Technical Optics Department here have carried out the test under my direction on similar lines. It is interesting to find that in the lens employed, the same as that used by Mr Kingslake, the effect of higher aberration in coma and astigmatism at 5° with the axis seems to be negligible, as shown by the straightness of the

focal line and the identity of the aberration coefficients derived from measurements on the zones of different radius.

This lens is corrected for spherical aberration and it is useless to compute the spherical aberration constants from results on two zones, but the correction of spherical aberration does not affect the validity of the test for the other aberrations. In point of actual fact, of course, the focal line is not accurately straight; the theory of primary aberration is only a close approximation within certain conditions, but its extreme utility is well illustrated in the present case.

I hope shortly to be able to apply these methods for the testing of oblique aberrations in microscope objectives.

Mr Kingslake has been a pioneer in the practical application of the theory and is to be congratulated both on the successful completion of the work and the clear manner of its description.

Mr W. D. Wright and Mr S. D. A. Hewitt: We have recently repeated the experiment, described by Mr Kingslake, in the Optics Department of the Imperial College, and we would like to call attention to one or two points connected with it. In the first place we do not agree with the estimate of a probable error of 1 per cent. An error of 3 or 4 per cent. seems more likely, considering the numerous sources of error involved. Our mean results differed from those obtained by Mr Kingslake by about 5 per cent., using the same lens at the same obliquity.

It is also necessary to point out that the lens was tested at 5° off the axis. At this obliquity the amounts of coma and astigmatism present, as shown by the star test, were quite intolerable; the lens could not be used for a greater angular field than about 3°, corresponding to 1·5° on each side of the axis. When the test is applied to more usual amounts of aberration, the accuracy falls very considerably, probably to something of the order of 10 or 15 per cent.

As the success of the experiment depends on the accurate placing of the holes in the diaphragm, it seemed desirable to test this accuracy. For this purpose we found it almost impossible to use a measuring microscope in the ordinary way, as even with a 3-inch microscope objective the field of view was too small, making it very difficult to align the diaphragm relative to the cross-wires of the microscope. (This would have to be done for each pair of holes.) This difficulty was overcome by using a short-focus telescope objective in place of the microscope objective, thus turning the instrument into a form of cathetometer. Measuring the diaphragm in this way, the error in positioning the holes was found to be approximately 0·5 per cent.

As regards the size of the holes, the agreement of Mr Kingslake's value with Hartmann's estimate of $\frac{1}{200}$ of the focal length is probably purely accidental. Presumably Hartmann's estimate is based on the distance of the photographic plate from the holes. When the separation between the inside and outside positions of the plates is small compared with the focal length of the objective, this distance will be sufficiently nearly the focal length of the objective for both plates. In the experiment described, however, the outer plate was nearly twice as far from the

diaphragm as the inner plate. Under these conditions the formula ($\frac{1}{200}$ of the focal length of the objective) for the size of hole can scarcely be said to hold.

With reference to Fig. 8, which represents the characteristic focal lines, it should have been pointed out that whilst the transverse aberration has been plotted in mm. the values for a are in cm. This produces a clearer diagram, but it must be realised that this also changes the slope of the focal lines with the principal ray. In reality the focal lines are nearly parallel to the principal ray: our results gave a value for θ (by the formula given, but not used, by Mr Kingslake) of $1^\circ\ 31'$.

We note that Mr Kingslake has made no use whatever of the tangential pairs of rays, represented in his diaphragm by T, $-T$ and t, $-t$. By calculating the intersections of these rays, additional values of a_2 and a_3 may be obtained. In this way we obtained quite distinct measures of a_2 and a_3 agreeing with our previous values within about 2·5 per cent.

Since the numerical values of the coefficients a_2 and a_3 depend upon the unit of length used in their determination, we suggest that the unit employed should be indicated.

Addenda to the results are obtainable from experimental values, and an interesting one might be quoted. The projections of the sagittal foci S, s (Fig. 8) upon the principal ray yield a measure of the spherical aberration. When the aberration is primary, this gives a method of determining a_1. The comparative freedom of this lens from spherical aberration, however, prevents any sensible value of a_1 from being computed, but the inability to do so is not without significance.

Mr J. Guild: What precautions are taken by the author to avoid errors due to the distortion of the photographic film which takes place between closely adjacent images? The cluster of points obtained in the Hartmann test would seem particularly well suited for this effect to become manifest.

Mr T. Smith: It will be useful to have the procedure followed in various applications of the Hartmann test on record in the *Transactions*. It would have been of value to include also the observations for the rays in the primary plane.

By the probable error in the aberration coefficients I presume is meant the probable error of the mean found from the ordinary expressions in the theory of errors. That value usually suggests a higher degree of accuracy than observations warrant, and it would be useful to record the difference between the mean and the extreme values calculated. I notice that the values for the comatic coefficients derived from the rays in the secondary planes differ considerably.

The behaviour and the properties of the focal line to which attention has been called are such as would be anticipated on going from the centre of the field outwards, for the points considered are all secondary foci, and the locus of central foci corresponding to them is the optical axis itself.

The proposal to detect the presence of higher order aberrations amounts merely to a definition of particular aberrations, as I show in a separate note*.

* T. Smith, *Trans. Opt. Soc.* 27 (1925–26) 242.

Tests to determine the coefficients mentioned were made previously by Chalmers.

Mr Kingslake: The method described by Mr Emsley for instructing students in the more familiar properties of aberrations with a large lens and various diaphragms is interesting. I would suggest the further experiment of isolating a single zone of the lens with a suitable ring-diaphragm and locating the characteristic focal line of the zone with a card held in the neighbourhood of the image, at an oblique angle to the principal ray. In this way a good idea of the physical reality of the existence of this characteristic focal line is obtained.

I hope Mr Emsley will find the formulae (1) given in the paper of value to students, as it is a simple expression for the primary aberrations, requiring little mathematical knowledge for its interpretation and use. Prof. Conrady is hoping to publish a book shortly containing many such formulae as this, which should go far to supply the want which Mr Emsley and others have felt. The symbols used in all Prof. Conrady's work are perfectly consistent and systematic; they are also chosen so as to be as far as possible self-explanatory, such as O.P.D. for "optical path difference," or L.Z.A. for "longitudinal zonal aberration." This leads to a lot of symbols, but it enables the student to realise at a glance what the equation involves. Similarly capital letters are used for all the data of a marginal ray, small letters referring to the corresponding data of a paraxial ray. Thus focal length is written small f, as it is a paraxial quantity, whereas the incidence height at a surface is written capital Y, as it is of course associated with a ray at finite aperture. It is the custom in many recent books on optics to use n for refractive index, as μ frequently enters the same formulae to represent magnetic permeability, or the micron ·001 mm. But since n is frequently used as a general symbol for any integer in summing series, Prof. Conrady prefers N for refractive index. μ is awkward also as it does not appear on the typewriter.

The discrepancies between my figures and those of Messrs Hewitt and Wright are very interesting. The ± 1 per cent. is, as Mr T. Smith suggests, the least square probable error of the average; but, by the definition of probable error, an error larger than this figure is as likely as an error smaller than this. The diaphragm used by these students was one having less holes than the one I used originally, and errors in the positions of the holes would be quite likely to account for some of the 5 per cent. Moreover all the details of the experimental method, such as fixing the diaphragm to the lens in a perfectly square and central position, are liable to slight errors. Practice in making Hartmann tests is very valuable, a beginner finding it at first almost impossible to repeat any results, but later it becomes quite possible with care. There is no real objection to performing the test at a greater obliquity than the one finally to be adopted, since coma is strictly proportional to H, and astigmatism to H^2; so that, as long as no secondaries are introduced, it is better to use a considerable obliquity, and reduce the results to the actual tilt required. If the experiment were performed at 1¼° tilt, there would only be a quarter the amount of coma, and one-sixteenth the amount of astigmatism;

but, since the experimental errors remain the same, the percentage errors would be increased by four and sixteen times respectively.

The size of hole is unimportant so long as it falls in the range of $\frac{1}{100}$ to about $\frac{1}{300}$ of the distance from the diaphragm to the plate; but, since this quantity varies for the two exposures, the mean value of about $f/200$ is a convenient figure to adopt. A little diffraction effect is frequently an advantage, especially if it is such as to produce tiny *rings* instead of dots on the photographs. It is found to be far simpler to set on such a ring than on a relatively large circular dot.

The unit employed in the values of a_2 and a_3 given in the paper is the centimetre, or rather 1/cm.^{2} for coma and 1/cm. for astigmatism, taking into account the dimensions of these coefficients. The dimensions of a_1 would similarly be 1/cm.^{3}

Mr Guild has pointed out a source of error which had never occurred to me before. However, from figures obtained by workers in the Kodak Research Laboratory*, it appears that two dots a few tenths of a mm. apart will displace each other by a few hundredths of a mm., but that above ·5 mm. separation no displacement is discernible. In my plates the nearest dots were separated by ·8 mm., so that I feel justified in neglecting the possibility of any displacement in this case. The matter is very interesting, and I am indebted to Mr Guild for having pointed it out, as it sets a lower limit on the distance from the focus at which the Hartmann photographs should be taken.

Several members have asked for the data of the tangential pairs $\pm T$ and $\pm t$. These were obtained from the plates at the time of the experiment, but were not included in the paper as they do not constitute *D*-pairs, and therefore need further formulae. But since for these pairs

$$TA_T = 3TA_S$$

they yield two further measures of a_2, namely ·001737 and ·001688. Also it can be shown that

$$LA_T - LA_S = 2a_3H^2,$$

whence for the two zones a_3 is found to be ·01316 and ·01462. The inclusion of these figures would not greatly affect the averages given, although they all tend to be rather low.

In reply to Mr T. Smith, the extreme values of a_2 were ·001816 for the pair ee' by equation (α), and ·001688 for the pairs $\pm aa'$ by equation (γ). This greatest value is 3·7 per cent. above the average, which implies an error of about 1 per cent. in the measurements on the plates. But since for this pair l_1 was only 1·5 mm., the *maximum* error in plate measurement appears to be about 15μ. It is interesting to note that the actual diameter of the dots being measured is about 100μ.

Mr Smith appears to have somewhat missed the significance of the characteristic focal line as being the locus of the foci of all the *D*-pairs *in a given zone*. For an axial pencil, light from all parts of a zone meets the axis at one point, the focus of that zone, so that the locus of central foci corresponding to the characteristic focal

* F. E. Ross, *The Physics of the Developed Photographic Image*, p. 165.

line is a single point on the optical axis. It is not, as Mr Smith surmises, the optical axis itself.

Mr Frank Robbins: Quite recently the Hartmann test has been applied to the 26·5-inch Grubb objective at Johannesburg (the largest south of the Equator) by Mr H. E. Wood, the Chief Assistant, on behalf of the Director, Dr R. T. A. Innes. The screen used was of cardboard and had 36 holes, each 1 inch in diameter, that is, $\frac{1}{491}$ of the focal length. It was the same screen as had been used by Dr Harold L. Alden to test the 26-inch photographic objective of the Yale University Southern Station on the Witwatersrand, South Africa; this fact ensures its suitability for the purpose. The resulting values of T ("der mittlere Durchmesser des geometrischen Zerstreuungsscheibchens" or more shortly, Hartmann's characteristic quantity or criterion) are 0·22 and 0·33 as shown in the accompanying table.

Hartmann's Criterion T

Johannesburg 26·5-inch Grubb objective. Focal length = 10700 mm. = scale 60.

Zone	Zone radius in inches (area)	Achernar = α Eridani Mag. 0·6. Spectrum B 5 Imperial S.S. made yellow sensitive		Canopus = α Argus Mag. −0·9. Spectrum F_0 Imperial S.S. *not* yellow sensitive	
		Scale reading	Scale reading minus foci mm.	Scale reading	Scale reading minus foci mm.
1	4·0 (16)	60·73	+0·73	64·49	+4·49
2	7·0 (33)	59·40	−0·60	64·64	+4·64
3	8·5 (23)	59·62	−0·38	65·13	+5·13
4	10·5 (38)	59·58	−0·42	64·33	+4·33
5	11·5 (22)	58·92	−1·08	64·12	+4·12
6	12·5 (24)	58·35	−1·65	62·84	+2·84

Exposure 120 secs. $T = 0·22$. Results $T = 0·33$.

Reference to *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, Band 15, Nr. 46, p. 103, will enable comparison to be made with the T values for twelve objectives from 8 to 40 inches in diameter and by five different makers. These values range from 0·16 for the Yerkes 40-inch to 3·04 for a Steinheil 80 cm. A perfect objective would have a zero value for T. The 24-inch objective at the Sproul Observatory, Swarthmore, U.S.A., gave values of 0·068 and 0·086. All this appears to justify the opinion of Mr H. E. Wood that, so far as can be judged from this criterion, the Johannesburg objective, while not perfect, is "pre-eminently good."

The pattern thrown by the screen on the plate is small. For example, the 30-inch common mirror at the Khedival Observatory, Helwan, gave a pattern only 3 mm. in diameter, "but this was found amply large enough for accurate measurement" by H. Knox Shaw in 1914. As regards the screen it is easy to see that the more holes it carries, the more exhaustive will be the test, but on the other hand the smaller the diameters of the holes, the more indistinct will be the images. Further

information on this subject can be obtained from a study of the work of Miss Marion R. Stearne published in the *Journal of the Franklin Institute*, vol. 178, No. 4 (Oct. 1914).

(Some of the preliminary negatives obtained while the 26·5-inch Grubb objective at Johannesburg was in course of adjustment, and also slides illustrating the various parts of the telescope, were shown at the meeting. Various other slides of general interest were also shown, including one of an address recently presented to Sir Howard Grubb by 18 well-known astronomers and mathematicians as a token of their high appreciation of the value of his services to astronomy.)

Mr Kingslake: I am very interested to hear this description of the Hartmann test being applied to a large telescope, but the photographs shown do not appear to be very clear or sharply defined. I should like to ask Mr Robbins whether the diaphragm was placed in front of or behind the lens.

The figures expressing the result of this test would be more interesting if they had been obtained with light of a fairly definite wave-length, for which purpose I would suggest placing a suitable filter about 1 cm. in front of the photographic plates during exposure, as in that position the effect of the parallel plate of glass on the spherical aberration would be negligible. If this were done, the dot-patterns on the plates should become quite sharp and clear. It would also be interesting if the tolerance for longitudinal aberration corresponding to the Rayleigh quarter-wave limit were included in the table for comparison.

Mr Robbins: The screen was placed between the stars and the objective and close to the latter. Dr Innes is still open to consider any other test—in fact in his last letter he says "but I have found another test for the object glass and this will be applied in the course of a few weeks." It is therefore certain he will welcome the remaining suggestions of Mr Kingslake when they reach Johannesburg some time in April.

Dr John S. Anderson: Before discussing some of the recent modifications of the Hartmann test perhaps I may be permitted to strike a personal note. I had the pleasure of studying astrophysics under Prof. Hartmann at Göttingen University before the War. He was one of my examiners for the degree and, in his capacity of Dean of the Faculty of Science, presided at the oral examination. I shall never forget his unfailing courtesy and his great kindness to me in all my dealings with him.

For many years the Hartmann test held the field as the most accurate quantitative method of measuring the axial aberrations of lenses—especially large lenses such as are used for astronomical purposes. The only serious rival in the field was the much older Foucault knife-edge method, but it lends itself more easily to the detection of the presence of aberrations than to their accurate measurement. The Hartmann method is very simple, but its chief drawback is, I think, that it is rather tedious to carry out. The extension of the method to microscope objectives was described to this Society some time ago by Prof Martin*, and in the present

* L. C. Martin, *Trans. Opt. Soc.* 23 (1921–22) 63.

paper Mr Kingslake has given us very interesting results which he has obtained from the application of the method to the case of oblique aberrations.

Some years ago the Hartmann test was thrown into the background by Mr Twyman's well-known application of interference methods to the testing of lenses. I think there is no doubt that in many respects the interferometer as developed by Mr Twyman is the most convenient instrument for measuring directly in terms of path differences the aberrations of lenses. The instrument is, however, very costly, and it is therefore desirable that alternative methods of test should be developed as thoroughly as possible.

In recent years there have been some interesting developments in the technique of the Hartmann method, and it may be worth while giving a brief sketch of them here. In the usual Hartmann method the dimensions of the holes are carefully chosen so that the images on the photographic plates are as free as possible from interference effects caused by the light from neighbouring holes. On the other hand the developments to which I have referred depend primarily on the formation of such interference effects. As Mr Smith points out in his note*, Chalmers† appears to have been the first to make use of this modification. He employed two narrow apertures in front of and close to the objective under test in order to produce an interference pattern as in Lord Rayleigh's interferometer. Spherical aberration was detected and measured by the displacement of the central band as the two slits, kept at a fixed distance apart, were traversed across the objective.

More recently Michelson‡, evidently not aware of Chalmers' work, suggested a similar method, one of the apertures being kept fixed on the optical axis of the objective and the other moved to different positions along a diameter of the objective.

Following on this suggestion Cotton§ and Merland‖ employed the system of two apertures separated by a fixed distance—the system used by Chalmers. As Merland points out, this modification in the technique overcomes two objections to the Michelson method, namely, that when the apertures are widely separated the fringes come very close together, and any lack of homogeneity in the air causes the fringes to be less steady owing to the separation of the interfering beams. Merland employed two methods of determining the phase differences in the emerging wave-front, (1) by observing the fringe systems directly and compensating the phase differences by means of a Jamin compensator, and (2) by photographing the fringes for various positions of the slit combination.

The most recent development of such methods was made by Gardner and Bennett¶ who employed a series of equidistant slits. Instead of placing this grating in front of the lens they placed it beyond the focal plane and obtained a photograph giving images of the slits with systems of interference fringes formed between them.

* T. Smith, *Trans. Opt. Soc.* 27 (1925–26) 242.
† S. D. Chalmers, *Proc. Opt. Convention*, 2 (1905) 156.
‡ A. A. Michelson, *Astrophys. Journ.* 47 (1918) 283.
§ A Cotton, *Physica*, 1 (1921) 274.
‖ A. Merland, *Rev. d'Opt.* 3 (1924) 401.
¶ I. C. Gardner and A. H. Bennett, *Journ. Opt. Soc. Amer.* 11 (1925) 441.

The aberrations of the lens can then be determined in terms of path differences from the measured displacements of the central fringes from the mid-way positions between the slit images. The method is a very simple one and possesses many advantages over the previous methods. One exposure is sufficient for determining the spherical aberration of a lens. The position of the focal plane of the lens does not need to be known accurately as in the other methods. The position of the focus for the various zones can be determined from the measurements of the plate and the known distance of the plate from the series of slits. It is claimed that an accuracy of approximately $\cdot 02\lambda$ in phase determination can be obtained.

I have recently made some preliminary tests of this method and, although it is too early yet to say much about it, there is reason to believe that it will lend itself easily to the measurement of oblique aberrations. In conjunction with interferometer tests it is hoped that this new method will yield interesting information with regard to a number of problems.

In this method there should be no trouble experienced from the distortion effect to which Mr Guild has referred, except perhaps in the case of the extreme fringes.

Gardner and Bennett in their paper give useful information with regard to the best sizes and separations of slits, but unfortunately do not say what kind of photographic plates they use. It is obvious that a very fine-grained plate should be used in order to get sharp interference fringes.

(Lantern slides illustrating the type of record obtained with the method of Gardner and Bennett were shown.)

Prof. L. C. Martin: Closely allied to the methods which Dr Anderson has described are those which have recently been employed by Ronchi.

NOTE ON CHALMERS' APPLICATIONS OF THE HARTMANN TEST

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MS. received, 17*th May*, 1926. *Read* (*in abstract*), 11*th March*, 1926.

ABSTRACT. Attention is called to Chalmers' application of the Hartmann test in 1904 to the determination of all relevant aberrations of photographic lenses. It is shown that the theory of these tests, whether with the photographic plates normal to the lens axis or normal to the chief ray, is readily constructed with the aid of the characteristic function. The limitations to and generalisations of Conrady's observation that rays from an axially centered circular zone have their secondary foci on a straight line are discussed. The method of testing lenses by means of interference fringes produced by two or more small apertures is claimed to have originated with Chalmers, whose description anticipated Michelson's by six years.

The Hartmann test for lenses attracted considerable attention in this country as well as on the Continent when it was first described, and trials were carried out by some members of this Society. In particular one of our Past-Presidents, S. D. Chalmers, whose activities covered so wide a field, proposed to use the method for the measurement of the aberrations of photographic lenses. In this project he was entirely successful, and the properties of a number of lenses were investigated over the whole of their fields. As in so many other directions, so in these tests also Chalmers' published papers fail to give an adequate idea of the extent of the work he carried out. He loved research, and gave up all his available time to it, so much so that he only spared time to write a description of his experiments or theoretical investigations when compelled by some reason or other to do so. In consequence of this very limited publication I think his memory has suffered somewhat, and I have noticed that even of the work that he did publish much has passed out of mind. I wish therefore now to recall the attention of the Society to his papers on testing lenses by such methods as we have been considering to-night, inadequate though they are as an account of his work on the Hartmann and allied tests.

The first paper was read before our Society just over 22 years ago, and is printed in the fourth volume of our *Transactions*. At that time Chalmers took the two photographs inside and outside focus on one plate, with the positions slightly displaced relatively to one another. A number of his photographs are reproduced in the paper, which only describes in outline the way in which the photographs are used. The absence of an adequate theoretical discussion in this paper is

probably due to the custom followed at that date of avoiding any but the very simplest mathematics in papers read before the Society. Until about the third year of the War the Society regarded the publication of much mathematical work in the *Transactions* as undesirable. A year later, 21 years ago this month, he described his methods more fully in a paper read before the Royal Photographic Society and published in the *Photographic Journal* for April, 1905. In this paper are reproduced records extending to 25 degrees from the axis. The eclipse of the outer parts of the aperture as the angle is increased is clearly seen in these photographs. One of the methods Chalmers used to deduce his measures of the aberrations is also given. From my knowledge of his work on this subject I wish to say that he investigated a number of methods, and that these were elaborated to the extent necessary to enable him to evaluate the aberrations of all orders of significance in the photographic lenses of that date. In some of his experiments the photographic plates were maintained in fixed planes normal to the axis for all angles of view, and the analysis of the aberrations was based upon the intersections of the rays with the image plane through the paraxial image as given by the characteristic function. The work of measurement on the plates was shortened in this method by taking some care that the two plates were at equal distances on either side of this image plane; one plate was then superposed on the other and the difference between the positions of corresponding image dots was measured.

Since Chalmers was in the habit of trying numerous variations in method, it is not unlikely that he tried also the slightly different form of test with the plates normal to the chief ray of the pencil under examination. I do not know whether in fact the notes he left include an examination of this arrangement, but I offer the following analysis of the problem, carried out with the help of the characteristic function, his favourite tool for optical research, as an indication that the theory would offer him no difficulties, whatever the order of the aberrations. The characteristic function V represents the length of the stationary optical path between a point (x, y, z) in the object space and a point (x', y', z') in the image space, when these two points are not conjugate to one another. In a medium of refractive index unity V changes in value by an amount equal to the displacement if this is along the ray passing through the two points, and does not change in value at all for small displacements normal to the ray. The fundamental property of the function, viz.

$$\left.\begin{aligned} L &= -\frac{\partial V}{\partial x}, \quad M = -\frac{\partial V}{\partial y}, \quad N = -\frac{\partial V}{\partial z} \\ L' &= -\frac{\partial V}{\partial x'}, \quad M' = \frac{\partial V}{\partial y'}, \quad N' = \frac{\partial V}{\partial z'} \end{aligned}\right\} \qquad \ldots\ldots(1),$$

where (L, M, N) and (L', M', N') are the direction cosines of this ray in the object space and in the image space respectively, at once follows.

The reference points in which we are interested in constructing this theory lie in the plane of the object and in the plane of the Hartmann stop. Since these are both normal to the axis, x and x' are constants, and V may be regarded as a function

of y, z, y', and z' only. Moreover, since the system of lenses is symmetrical about the x axis, V is actually a function of the three variables α, β, and γ given by

$$\alpha = y^2 + z^2, \quad \beta = yy' + zz', \quad \gamma = y'^2 + z'^2 \qquad \ldots\ldots(2).$$

Instead of taking y and z as the coordinates of the actual object point, they may be the constant multiples of these coordinates which would be the image coordinates if the lens were entirely free from aberrations. They denote these ideal image coordinates in what follows, so that we have to consider dimensions in the image space alone.

In terms of the new variables

$$\left.\begin{aligned} M' = V_{y'} &= yV_\beta + 2y'V_\gamma \\ N' = V_{z'} &= zV_\beta + 2z'V_\gamma \end{aligned}\right\} \qquad \ldots\ldots(3),$$

and since $y' + \rho M', \quad z' + \rho N'$

are two of the coordinates of a point on the ray distant ρ from (y', z'), these coordinates may be written

$$\left.\begin{aligned} \eta &= y\rho V_\beta + y'(1 + 2\rho V_\gamma) \\ \zeta &= z\rho V_\beta + z'(1 + 2\rho V_\gamma) \end{aligned}\right\} \qquad \ldots\ldots(4).$$

We may assume that the object point, and consequently its ideal image point also, lie in the plane $z = 0$ without loss of generality. The secondary image is then given by $\zeta = 0$. It at once follows from (4) that the distances ρ and η of this image from (y', z') and the axis respectively are given by

$$\rho = -\frac{1}{2V_\gamma} \qquad \ldots\ldots(5)$$

and

$$\eta = y\rho V_\beta = -\frac{yV_\beta}{2V_\gamma} \qquad \ldots\ldots(6)$$

respectively. These equations may be used directly when the plates are normal to the axis of the lens.

When the plate is normal to the chief ray we have to find the distances of the secondary focus along and perpendicular to the chief ray from these expressions. Since the coordinates of the image point referred to the main axes are $(\rho L', \eta, 0)$, the coordinates referred to the chief ray are $(\Xi, H, 0)$, where

$$\Xi = \rho L'l' + \eta m',$$
$$H = \eta l' - \rho L'm',$$

and $(l', m', 0)$ are the direction cosines of the chief ray, obtained by inserting $y' = z' = 0$ in (3). Thus by (6)

$$\Xi = -\frac{L'l' + \alpha V_\beta v_\beta}{2V_\gamma} \qquad \ldots\ldots(7)$$

and

$$H = -\frac{yL'l'}{2V_\gamma}\left(\frac{V_\beta}{L'} - \frac{v_\beta}{l'}\right) \qquad \ldots\ldots(8),$$

where, in harmony with the notation just adopted, v_β is the value of V_β when $\beta = \gamma = 0$. It will be observed that the term v_β/l' in (7) merely serves to remove from V_β/L' the terms which are independent of β and γ.

Though equations (5) to (8) furnish a complete theoretical foundation for the Hartmann test as applied to secondary images, it will perhaps not be without interest to carry the investigation a step further. Consider first an optical system which is entirely free from aberrations. The length of the optical path from a given object to its image (x, y, z) has the same value for all paths. The part of this path between the point (x', y', z') of the stop and the image point is

$$r = \{(x - x')^2 + (y - y')^2 + (z - z')^2\}^{\frac{1}{2}} = \{(x - x')^2 + \alpha - 2\beta + \gamma\}^{\frac{1}{2}}.$$

Thus the length of path between the object point and the stop is

$$b - \{(x - x')^2 + \alpha - 2\beta + \gamma\}^{\frac{1}{2}}.$$

The value of b will vary with the object point, and is thus a function of α only. In the absence of aberrations then the characteristic function has the value

$$\phi(\alpha) - \{(x - x')^2 + \alpha - 2\beta + \gamma\}^{\frac{1}{2}}.$$

If the value of the path differs from this, aberrations are present. Thus in general

$$V = \phi(\alpha) - \{(x - x')^2 + \alpha - 2\beta + \gamma\}^{\frac{1}{2}} + U \qquad \text{.....(9),}$$

where U is a function of x', α, β, and γ, the coefficients of which measure the various aberrations present in the system.

This expression is not strictly the characteristic function itself, though equal to it in value. For it has been assumed that (x, y, z), instead of identifying a general object point, identifies a point in a particular plane. We are not therefore at liberty to ascribe to the expression on the right of (9) the properties of the characteristic function which depend upon differentiation with respect to x, y, and z. This objection does not apply to differentiation with respect to x', y', and z', and it will be observed that in deriving equations (4) to (8) differentiation with respect to y' and z' alone has been used. For substitution in these equations we may therefore treat (9) as the real characteristic function without introducing error. Moreover, as we are dealing with a stop in a fixed plane normal to the lens, x' may be regarded as a constant, and the value of L' will then be derived from those of M' and N'. We find at once that V_β, $-2V_\gamma$, and $\dfrac{L'}{x - x'}$ all have as their non-aberrational parts $1/r$. Thus if we stop short at first order aberrations, for which we may put

$$U = -\frac{1}{8f^3}\{\sigma_0\alpha^2 - 4\sigma_1\alpha\beta + 4\sigma_2\beta^2 + 2(\sigma_2 + \omega)\alpha\gamma - 4\sigma_3\beta\gamma + \sigma_4\gamma^2\}$$
$$\text{.....(10),}$$

where f is the focal length of the lens and the aberration coefficients are pure numbers, we find on retaining only the first powers of α, β, γ,

$$\Xi = r_0 - \frac{(x - x')^2}{2f^3}\{(\sigma_2 + \omega)\alpha - 2\sigma_3\beta + \sigma_4\gamma\} \qquad \text{.....(11),}$$

$$\mathrm{H} = \frac{y(x - x')}{2f^3}\{-2\sigma_3\beta + \sigma_3\gamma\} \qquad \text{.....(12),}$$

which only differ from Conrady's expressions quoted by Kingslake* in distinguishing between $(x - x')$ and f. r_0 of course denotes the distance from the centre of the Hartmann diaphragm to the ideal image point when aberrations are absent.

It ought to be observed that these expressions may not be admissible for the reduction of observations when the aperture is large or the obliquity is appreciable, even though higher order aberrations are absent and the products and squares of the first order coefficients are negligibly small. Under these conditions, if r and r_0 are the distances of the points (y', z') and $(0, 0)$ of the Hartmann stop from the ideal image point, the exact expressions are, from equations (5) and (6),

$$\xi = x - x' + \frac{r}{x - x'}\{2r^2U_\gamma - (\alpha - \beta)(U_\beta + 2U_\gamma)\} \quad \ldots\ldots(13),$$

$$\eta = y + yr(U_\beta + 2U_\gamma) \quad \ldots\ldots(14),$$

and from (7) and (8)

$$\Xi = r_0 + \frac{r}{r_0}\{2r^2U_\gamma + \beta(U_\beta + 2U_\gamma)\} \quad \ldots\ldots(15),$$

$$\mathrm{H} = \frac{y}{r_0(x - x')}\{r^3U_\beta - r_0{}^3u_\beta + r(\beta - \gamma)(U_\beta + 2U_\gamma)\} \quad \ldots\ldots(16).$$

These equations illustrate how readily the theory of such tests may be developed with the aid of the characteristic function. If the products of first order aberrations are negligible these expressions give all that is required for the analysis of second order aberrations. If the products are not negligible the only modification necessary is to include the products of the U's in substituting in the fundamental expressions (5) to (8). This offers no difficulty. It should be noted that by investigating the theory in this way the formulae are established for systems including refracting surfaces of any shape symmetrical about the axis.

A few words may be said on Conrady's very interesting observation on the focal line. It is curious that this particular correspondence between the circular zone of the lens and the straight focal line has not been noted before, although more general theorems of this type were demonstrated long ago. If we wish to be quite precise we must note that the line is not quite straight even in the absence of all higher order aberrations†; equation (6), for instance, shows that there will be departures from straightness due to the products of first order coefficients. Indeed we can go farther and show that the diaphragm locus of points which gives a straight focal line is not precisely a circle even when the products of first order aberrations as well as higher order aberrations are neglected. For from (13) and (14) it is obvious that any linear relation between ξ and η involves at least the squares and products of β and γ. If we seek the simplest curve to which the focal line conforms when the rays meet the diaphragm in a circle, we must evidently take the centre of the circle at the same distance from the axis as the ideal image point. The value of r is then constant for all points of the circle, and the focal

* R. Kingslake, *Trans. Opt. Soc.* 27 (1925–26) 222, equation (1).

† It is assumed that separate terms in V correspond to distinct aberrations.

line is a parabola, with its axis parallel to the axis of the lens, whose latus rectum is equal to

$$\frac{a r r_0}{f^3}(\sigma_2 - 2\sigma_3 + \sigma_4).$$

If the amounts of first order astigmatism, coma, and spherical aberration are such that

$$\sigma_2 - 2\sigma_3 + \sigma_4 = 0,$$

the focal line becomes straight and is parallel to the axis of the lens.

Whenever it is permissible to neglect powers of β and γ above the first in (13) and (14), the rays meeting the diaphragm in *any* circle will form a straight focal line. We need only consider the case when the square of α is also negligible; the equation to the focal line corresponding to the circle of radius a with its centre a distance ky from the axis is then

$$\begin{vmatrix} \sigma_4 & \sigma_3 & (\sigma_2 + \omega)\,\alpha + \dfrac{(\xi - x + x')\,2f^2}{(x - x')^2} \\ \sigma_3 & \sigma_2 & \sigma_1\alpha + \dfrac{\{\xi y - \eta\,(x - x')\}\,2f^2}{y\,(x - x')^2} \\ 1 & k & k^2\alpha - a^2 \end{vmatrix} = 0,$$

showing that the focal lines for any series of concentric circles are parallel. If the angle made by the chief ray with the axis is ψ, the inclination of the focal line to the axis is θ, where

$$(\tan\theta - \tan\psi)(\sigma_3 - k\sigma_4) + \tan\psi\,(\sigma_2 - k\sigma_3) = 0.$$

According to this equation, if the lens is aplanatic the focal line is normal to the lens axis whatever k and a may be. If both astigmatism and coma are absent, the focal line is parallel to the chief ray. If the lens is free from astigmatism and spherical aberration,

$$\tan\theta = (1 + k)\tan\psi,$$

irrespective of the amount of coma present. In the absence of astigmatism, coma, and spherical aberration the focal line of course degenerates to a point and no meaning can be attached to direction. The distance of any focal line from the focus corresponding to the centre of the circular zones is proportional to the area of the circle and to $\sigma_2\sigma_4 - \sigma_3^2$. If $\sigma_2\sigma_4 - \sigma_3^2 = 0$, the whole aperture will form a single focal line. In the special case $\sigma_2 = \sigma_3 = \sigma_4$ the whole aperture forms a single focal line parallel to the lens axis. From the more exact expressions (13) and (14) under these conditions η is constant for the circle with $k = 1$, and the variation of ξ is governed by a small term in $\alpha\beta$. If $\sigma_1 - \omega = \sigma_2 = \sigma_3 = \sigma_4$, the rays from the whole zone pass through a single point.

The projection of the focal line on the axis is equal to

$$\frac{2r_0^3 ay}{(x - x')f^3}(\sigma_3 - k\sigma_4).$$

Thus if the circle is centered on the axis the length of the line measures the coma,

and if not centered on the axis it measures the spherical aberration in the absence of coma. Corresponding interpretations of other projections offer no difficulty. Enough has been said to show that these focal lines, particularly when the centres of the diaphragm zones are not limited to the axis, are of special interest when lenses are examined on the Hartmann principle.

The examination of this problem has led me rather far from my main theme. One word more, however, must be said in conclusion on those tests allied to the Hartmann test in which two or more apertures are arranged to give strong interference bands, the positions of which are measured for the numerical evaluation of the aberrations. In recent years such methods of testing lenses have been explored in much detail. Judging from references made in papers describing various forms of these tests, the original proposal to use this principle is generally attributed to Michelson. Michelson's paper* on the subject appeared in June 1918, and it appears that the credit is really due to Chalmers, who gave a preliminary description in a paper read before the Optical Convention of 1912. Further work on the subject, to which Chalmers gave much thought, for he fully realised its importance, was interrupted by the War, and later terminated by his untimely death. Out of regard for his memory I offer this note in the hope that it will help to secure wider recognition of Chalmers' work in initiating and developing these new methods of testing optical instruments.

* "On the Correction of Optical Surfaces." *Astrophys. Journ.* 47 (1918) 283.

THE DISTRIBUTION OF LIGHT IN ELEMENTARY OPTICAL IMAGES

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ABSTRACT. The paper presents the results of a series of calculations on the distribution of light near the "star focus" of a centered lens system in the following cases:

(*a*) Freedom from aberration.
(*b*) Primary spherical aberration.
(*c*) Zonal spherical aberration.

In the two latter cases the condition chosen is that when the least residual phase differences amount to $\pi/2$. The characteristic extra-focal effects are determined, and the effect of primary spherical aberration on resolving power is discussed. In the case of zonal aberration the greatest axial intensity is not found at the focus giving least phase residuals. In both cases of aberration a concentration closely resembling in some respects the "Airy" disc characteristic of zero aberration is found. These results are an extension of those previously obtained by Conrady and Buxton.

Introduction

The necessary starting-point in the systematic study of the distribution of light near the focus of a centered lens system is the investigation of the most elementary type of image, the "star disc," which results when the angular subtense of a distant axial source is indefinitely diminished. A number of theoretical investigations have been carried out by Airy (1), Rayleigh (2), Lommel (3), Strehl (4), Conrady (5), Buxton (6), Steward (7), the writer (8), and others. The writer has studied the subject experimentally (9), as have also Mrs C. H. Griffiths (10) and Miss H. G. Conrady (11). There are, however, a number of optical problems of importance, such as the question of chromatic aberration, which require both theoretical and practical study, and for purposes of this kind more detailed and systematic information regarding the light distribution near an axial star image is called for. To this end a series of calculations have been carried out, in order to extend and check the information already given by Conrady and Buxton. The method used was that already described by Conrady. The purpose of this paper is to present the results obtained and to comment shortly upon them.

A short explanation of the symbols used will facilitate the use of the results. In Fig. 1, *APCBD* represents a zone, of radius r, of the usual spherical reference surface with its centre at F, and OF is the axis. The point F' is at a distance K

from F, and FF' is perpendicular to the axis. The lines AB and CD, which intersect in O, are perpendicular diameters of the zone, DC being parallel to FF'. Let P be a point on the zone such that OP makes an angle E with the radius OA which is perpendicular to FF'. Then, writing PF as f,

$$PF' = f - \frac{Kr \sin E}{f} \qquad \ldots\ldots(1),$$

provided that K is so small that its square may be neglected, and provided that r is small in comparison with f. The ring may represent the boundary of a spherical wave-front having its centre of curvature or focus at F. In this case all disturbances

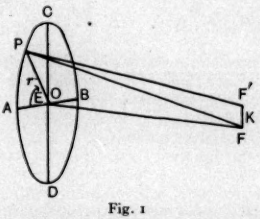

Fig. 1

from the wave elements have the same phase at the point F, but at the point F' there will be a difference W in phase between disturbances arriving from A or B and from C which is given by

$$W = \frac{360^\circ}{\lambda} \cdot \frac{Kr}{f} \qquad \ldots\ldots(2),$$

expressing the phase angle in degrees, and writing λ for the wave-length. While the disturbance from C leads in phase, that from D will evidently lag by an equal amount.

For given values of r and f, W is proportional to K, and is a convenient unit for denoting perpendicular displacements from the axis, but the scale will change in passing along the axis owing to the variation of f. The "W values" in the numerical results given below are all taken with regard to the radius of the diaphragm which limits the wave-front.

Under the limitations for which the formula (1) is valid, the resultant phase at F' for the whole ring will clearly be identical with that of the disturbances from A and B, but the amplitude of the resultant disturbances will depend on r, K, and f.

Now let the point F be taken at a small distance δf measured along the optical axis from the wave centre. The path difference thus introduced between the disturbances arriving at F from the axial and marginal regions of the wave-surface will be given to a sufficient approximation by

$$\delta p = (\text{increase of axial path} - \text{increase of marginal path}) = \frac{\delta f}{2}\left(\frac{r}{f}\right)^2 \qquad \ldots\ldots(3).$$

Thus the phase variation of the resultant disturbances from different zones is

proportional to the square of the radius, and this holds for the *phases* of the zonal resultants at F', but at this point the amplitudes of the resultants will also vary.

In the case of an axial image point, the phase difference ϕ for the disturbance from a zone of radius r may be represented by a series of even powers of r,

$$\phi = c_1 r^2 + c_2 r^4 + c_3 r^6 + \ldots.$$

As just mentioned, the term in r^2 corresponds to the effect of a small change of focus; the terms in r^4, r^6, and higher even powers represent the phase differences of disturbances from various zones in the spherical reference surface, having its centre at the paraxial focus, which are due to spherical aberration. Primary spherical aberration gives the term in r^4 only, the coefficients of higher powers of r being zero in such a case.

The method of integration used for the present work has been fully explained elsewhere(5). Briefly, it is equivalent to finding by mechanical quadratures the value of the expression

$$I = \text{const.} \times \left[\left\{ \int_{r=0}^{r=1} J_0(\alpha r) \cos\phi \, d(r^2) \right\}^2 + \left\{ \int_{r=0}^{r=1} J_0(\alpha r) \sin\phi \, d(r^2) \right\}^2 \right],$$

where $\alpha = 2\pi K/\lambda r$ and ϕ has the meaning given above. The radius of the marginal zone is taken as unity. It has not been judged advisable to carry out the work for W values greater than 600°, as the probable error of the results begins to be serious owing to the necessary approximations made in the use of tables, etc. In addition, the final results depend on the sums and differences of small quantities which may suffer some inaccuracy. The errors in the tables given should not, however, exceed 5 units in the last decimal place, and will usually be less than this.

Cases chosen for Integration

(1) The first integrations were performed for the case

$$\phi = c_1 r^2 \qquad \ldots\ldots(4),$$

which is the case of simple changes of focus. The radius of the marginal zone is taken as unity, and c_1 is given the values 0°, 45°, 90°, ... 540°, the light intensities being calculated for $W = 0°$, 40°, 80°, ... 600° in all the corresponding planes. These results (as in the other cases) include, and correct in some details, some of those already calculated by Conrady and Buxton. There will evidently be symmetry on each side of the focus in the sense described above, that is, when distances from the axis are expressed in terms of W. The results are given in Table I and Fig. 2; the arrangement of the curves will be obvious on inspection, the base for each curve being taken one step higher in consecutive cases. The vertical scale is given for the lowest curve.

(2) The next case is that of primary spherical aberration,

$$\phi = c_1 r^2 + c_2 r^4 \qquad \ldots\ldots(5).$$

The maximum radius is again assumed to be unity, and c_2 is given the value 360°,

Table I. *Table of light intensity near the focus. No aberration present.* $\phi = c_1 r^2$. (Radius of marginal zone is unity.)

W of marginal zone	0°	40°	80°	120°	160°	200°	240°	280°	320°	360°	400°	440°	480°	520°	560°	600°
c_1 in degrees																
0	1·000	·8845	·6008	·2953	·0871	·0066	·0042	·0164	·0145	·0046	·0000	·0021	·0041	·0027	·0004	·0002
45	·9493	·8400	·5710	·2828	·0876	·0132	·0108	·0206	·0162	·0052	·0004	·0025	·0044	·0029	·0005	·0002
90	·8102	·7175	·4890	·2477	·0884	·0302	·0283	·0315	·0212	·0076	·0023	·0041	·0055	·0031	·0005	·0005
135	·6151	·5447	·3740	·1986	·0886	·0528	·0511	·0454	·0274	·0105	·0049	·0067	·0074	·0043	·0012	·0009
180	·4051	·3590	·2498	·1437	·0853	·0721	·0708	·0574	·0336	·0153	·0100	·0112	·0102	·0058	·0021	·0018
225	·2215	·1966	·1404	·0935	·0776	·0921	·0813	·0634	·0381	·0212	·0169	·0173	·0140	·0079	·0036	·0032
270	·0900	·0802	·0617	·0542	·0654	·0760	·0794	·0616	·0393	·0269	·0240	·0209	·0180	·0104	·0057	·0055
315	·0194	·0177	·0180	·0295	·0504	·0668	·0666	·0525	·0374	·0314	·0284	·0291	·0216	·0132	·0089	·0085
360	·0000	·0004	·0047	·0174	·0353	·0475	·0474	·0393	·0330	·0330	·0349	·0320	·0238	·0144	·0124	·0121
405	·0117	·0106	·0098	·0141	·0221	·0279	·0280	·0256	·0266	·0314	·0345	·0372	·0239	·0176	·0157	·0156
450	·0324	·0287	·0201	·0150	·0131	·0131	·0132	·0149	·0201	·0267	·0301	·0276	·0220	·0185	·0183	·0186
495	·0458	·0405	·0284	·0162	·0086	·0056	·0056	·0085	·0144	·0204	·0229	·0214	·0185	·0180	·0194	·0196
540	·0453	·0397	·0277	·0155	·0073	·0048	·0048	·0067	·0104	·0156	·0143	·0138	·0143	·0146	·0179	·0162

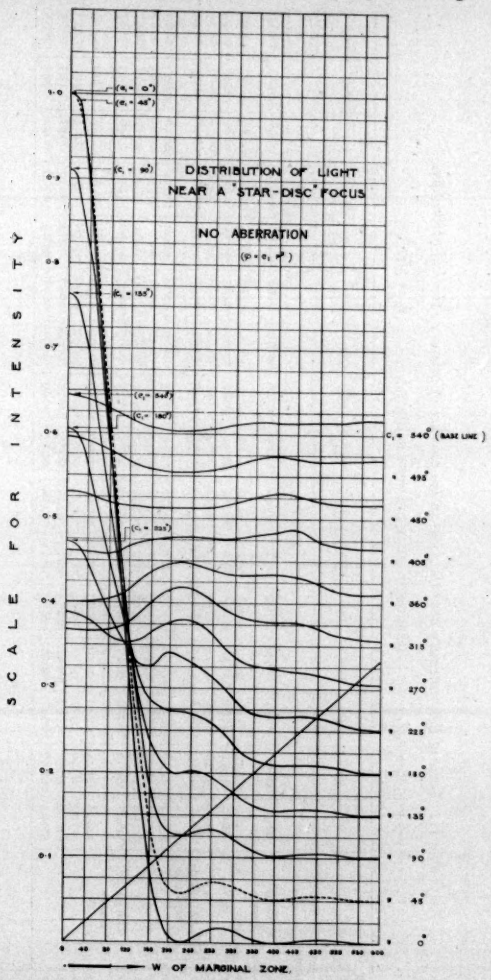
DISTRIBUTION OF LIGHT
NEAR A "STAR-DISC" FOCUS
NO ABERRATION
SCALE FOR INTENSITY
W OF MARGINAL ZONE.

Fig. 2

Table II. *Table of light intensity near the focus in the presence of primary spherical aberration.*

$\phi = c_2 r^4 - c_1 r^2$. (Radius of marginal zone is unity.) $c_2 = 360°$.

W of marginal zone		0°	40°	80°	120°	160°	200°	240°	280°	320°	360°	400°	440°	480°	520°	560°	600°
c_1 in degrees																	
0	Paraxial focus	·0890	·0890	·0874	·0820	·0702	·0527	·0348	·0221	·0171	·0169	·0161	·0136	·0099	·0096	·0102	·0094
90		·1462	·1433	·1348	·1207	·0997	·0723	·0436	·0219	·0127	·0130	·0156	·0149	·0114	·0083	·0078	·0080
180		·3650	·3371	·2665	·1831	·1105	·0597	·0282	·0122	·0082	·0116	·0155	·0152	·0104	·0060	·0046	·0042
270		·6597	·5932	·4255	·2371	·0950	·0236	·0042	·0069	·0123	·0151	·0150	·0121	·0071	·0032	·0023	·0034
360	Least phase residuals	·8001	·7077	·4822	·2351	·0683	·0062	·0100	·0265	·0290	·0197	·0097	·0044	·0023	·0018	·0025	·0032
450		·6597	·5746	·3862	·1768	·0522	·0289	·0570	·0761	·0625	·0220	·0070	·0002	·0034	·0058	·0040	·0009
540		·3650	·3092	·1820	·0727	·0353	·0614	·0945	·0933	·0601	·0248	·0091	·0101	·0132	·0105	·0045	·0006
630		·1462	·1163	·0530	·0121	·0218	·0615	·0879	·0792	·0504	·0289	·0269	·0292	·0296	·0211	·0077	·0027
720	Marginal focus	·0890	·0692	·0280	·0017	·0075	·0302	·0433	·0381	·0282	·0287	·0387	·0450	·0342	·0246	·0143	·0117
810		·0998	·0825	·0440	·0118	·0007	·0039	·0055	·0062	·0083	·0194	·0322	·0370	·0282	·0236	·0208	·0226
900		·0770	·0646	·0371	·0132	·0034	·0031	·0030	·0008	·0011	·0097	·0131	·0158	·0152	·0162	·0208	·0253
990		·0276	·0221	·0112	·0050	·0073	·0123	·0128	·0078	·0024	·0004	·0007	·0014	·0030	·0071	·0127	·0165
1080		·0083	·0057	·0012	·0006	·0056	·0110	·0122	·0087	·0049	·0031	·0026	·0025	·0008	·0022	·0034	·0055

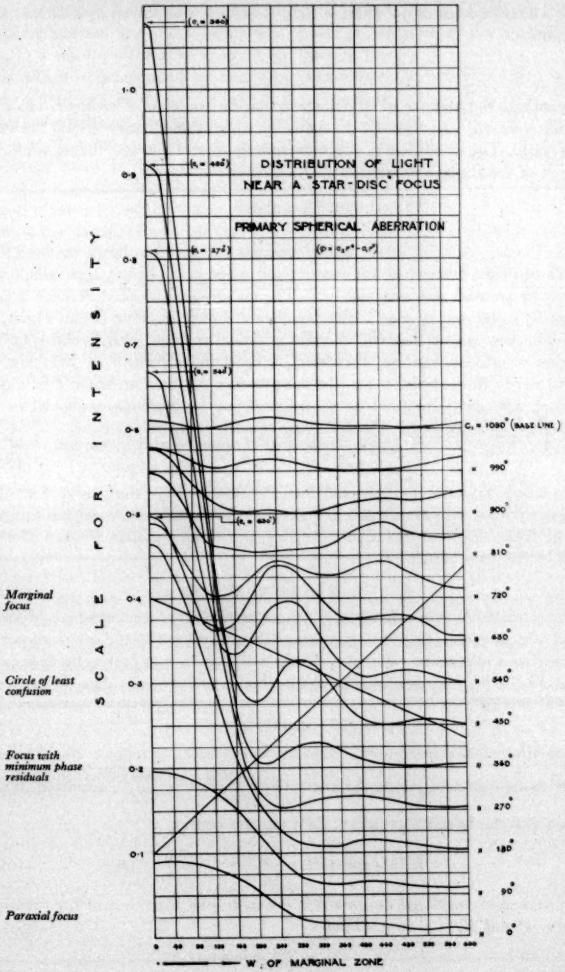
DISTRIBUTION OF LIGHT
NEAR A "STAR-DISC" FOCUS
PRIMARY SPHERICAL ABERRATION
SCALE FOR INTENSITY
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
Marginal focus
Circle of least confusion
Focus with minimum phase residuals
Paraxial focus
C₁ = 1080° (BASE LINE)
990°
900°
810°
720°
630°
540°
450°
360°
270°
180°
90°
0°
W, OF MARGINAL ZONE

Fig. 3

while c_1 varies in steps of 90° from − 1080° to 0°. It was shown by Conrady that the condition

$$\frac{d\phi}{dr} = 0 \qquad \text{.....(6)}$$

corresponds to the absence of lateral aberration for the zone of radius r, i.e. a ray from this zone will pass through the focus for which the equation giving the value of ϕ is valid. The condition $c_1 = 0$ corresponds to the paraxial focus, while the condition of the absence of angular aberration for the margin,

$$\left(\frac{d\phi}{dr}\right)_{r=1} = 0 \qquad \text{.....(7),}$$

gives $c_1 = -2c_2$, or $c_1 = -720°$ in our case. It can also be shown that the least residuals of phase differences are encountered when $c_1 = -360°$, that is half-way between the paraxial and marginal foci. The outstanding phase difference is then numerically equal to $c_2/4 = 90°$. The results are shown in Table II and Fig. 3.

(3) The last case is that one of great practical interest when zonal spherical aberration is present, but the aberration is kept under control by bringing the marginal ray to the paraxial focus. In the presence of a certain amount of higher aberration this can be ensured by the trigonometrical methods employed in the design of an optical system. The following equation may be assumed,

$$\phi = c_1 r^2 + c_2 r^4 + c_3 r^6 \qquad \text{.....(8).}$$

The equation valid for the paraxial focus is obtained by putting $c_1 = 0$. The condition (7) must then be fulfilled in order that the paraxial and marginal foci may coincide, which leads to the equation $c_2 = 3/2c_3$. The equation (8) can then be written in the form (writing m for c_1/c_3)

$$\phi = r^2 c_3 \{(r^2 - \tfrac{3}{4})^2 + (m - \tfrac{9}{16})\} \qquad \text{.....(9),}$$

which is of assistance in searching for the focus involving the least residuals of phase. If a trial is made of the value $m = \frac{9}{16}$, it is seen that the value of ϕ at the corresponding focus reaches a maximum value of $c_3/16$ for $r = \frac{1}{2}$ and $r = 1$, and is clearly zero for $r = 0$ and $r = \sqrt{\frac{3}{4}}$. The use of a graphical method is useful to elucidate the problem. From the expression

$$\phi = c_3 r^6 - \tfrac{3}{2} c_3 r^4 + c_1 r^2,$$

and remembering that lateral aberration

$$= \text{(focussing distance)} \times \text{(angular aberration)} = \text{const.} \times \left(\frac{d\phi}{dr}\right), \text{ approximately,}$$

it is seen that the lateral aberration (TA') may be written

$$TA' = \text{constant} \times \left(r^5 - r^3 + \frac{mr}{3}\right) \qquad \text{.....(10).}$$

Taking successive fractional values of r it is easy to plot the values of the functions $X_1 = r^5 - r^3$ and $X_2 = -mr/3$. Then

$$X_1 - X_2 = TA'/\text{constant.}$$

The functions X_1 and X_3 are plotted in Fig. 4. The horizontal intercept between the curve and the line at any ordinate is clearly proportional to the lateral aberration for a ray from the zone in question, taken at the focus which is determined by the slope of the line. Then a line can be found for which the maximum transverse intercepts are equal and opposite. M and N represent these intercepts in Fig. 4, and in this case it is easily found by trial that $m/3 = c_1/3c_3 = \cdot 108$ or $c_1 = \cdot 324c_3$. In this way the position of the circle of least confusion is approximately determined with respect to the paraxial focus.

The other focus of interest is represented by

$$m = 9/16, \text{ i.e. } m/3 = 0{\cdot}187.$$

The corresponding line is also plotted in the diagram.

From the proportionality between lateral aberration and the differential coefficient $d\phi/dr$ it is evident that we may write

$$\phi r_1 = \text{constant} \times \int_{r=0}^{r=r_1} (TA')\, dr.$$

The meaning of this equation is seen on reference to Fig. 4. The integral is the area intercepted between the line and the curve up to the ordinate $r = r_1$, and for $m/3 = \cdot 187$ there are three numerically equal intercepts A, B, and C. However, A and C must be considered positive, while B is negative, so that the value of the integral (as already found) reaches the same highest value for $r = \frac{1}{2}$ and $r = 1$, and vanishes for $r = 0$ and $r = \sqrt{\frac{3}{4}}$. It also appears immediately that the slope of the line cannot be altered without enlarging one or more of the areas A, B, and C, and therefore increasing the residual of phase difference between disturbances arriving from certain zones. Hence the greatest numerical phase residual is $c_3/16$.

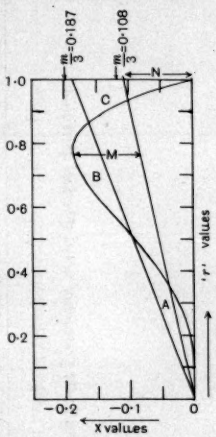

Fig. 4

An interesting case arises when $c_3/16 = 90°$. This gives $c_3 = 1440°$ and $c_2 = 2160°$. The focus involving least phase residuals is represented by $c_1 = 810°$, and that for the circle of least confusion by $c_1 = 467°$ (approximately). Integrations have been carried out for a series of values of c_1 from 630° to 1170° in steps of 45°. The results are given in Table III and Fig. 5.

Table III. *Distribution of light near the focus in the presence of zonal spherical aberration.*

$\phi = c_3 r^6 - \frac{3}{2} c_3 r^4 + c_1 r^2$. (Radius of marginal zone is unity.) $c_3 = 1440°$

W of marginal zone	0°	40°	80°	120°	160°	200°	240°	280°	320°	360°	400°	440°	480°	520°	560°	600°
c_1 in degrees																
630	·1076	·0960	·0744	·0676	·0811	·0936	·0818	·0473	·0142	·0038	·0142	·0263	·0258	·0155	·0041	·0057
675	·2538	·2251	·1606	·1049	·0818	·0784	·0671	·0394	·0115	·0030	·0140	·0270	·0272	·0163	·0056	·0026
720	·4315	·3822	·2650	·1485	·0792	·0554	·0456	·0283	·0089	·0032	·0134	·0258	·0265	·0160	·0048	·0007
765	·6053	·5354	·3663	·1898	·0751	·0320	·0245	·0186	·0079	·0051	·0121	·0225	·0233	·0145	·0042	·0000
810	·7325	·6484	·4414	·2202	·0711	·0135	·0091	·0136	·0099	·0057	·0097	·0170	·0187	·0124	·0039	·0003
855	·7938	·7023	·4767	·2338	·0683	·0049	·0043	·0155	·0152	·0085	·0067	·0108	·0120	·0088	·0035	·0009
900	·7784	·6888	·4694	·2285	·0662	·0063	·0095	·0237	·0237	·0128	·0051	·0044	·0060	·0052	·0026	·0012
945	·6967	·6166	·4185	·2062	·0639	·0144	·0214	·0360	·0336	·0187	·0058	·0012	·0017	·0023	·0015	·0009
990	·5716	·5058	·3439	·1723	·0600	·0249	·0350	·0478	·0436	·0261	·0095	·0014	·0000	·0006	·0006	·0004
1035	·4307	·3813	·2600	·1333	·0537	·0331	·0455	·0572	·0517	·0340	·0160	·0052	·0014	·0009	·0007	·0003
1080	·2974	·2634	·1792	·0952	·0447	·0362	·0498	·0607	·0567	·0413	·0238	·0115	·0054	·0031	·0027	·0011
1125	·1880	·1665	·1142	·0623	·0341	·0338	·0476	·0583	·0572	·0458	·0310	·0187	·0111	·0075	·0054	·0034
1170	·1106	·0980	·0676	·0379	·0234	·0268	·0398	·0510	·0534	·0471	·0356	·0244	·0165	·0125	·0100	·0071

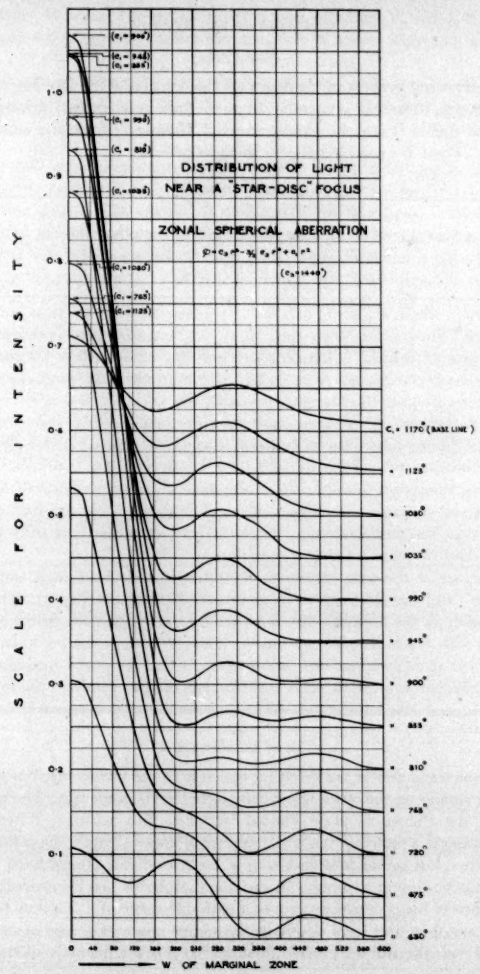
DISTRIBUTION OF LIGHT
NEAR A "STAR-DISC" FOCUS
ZONAL SPHERICAL ABERRATION
SCALE FOR INTENSITY
W OF MARGINAL ZONE

Fig. 5

Discussion of the Results

Ray Paths

It is interesting to show in the diagrams the position of the limiting rays. Let k be the distance, measured perpendicular to the axis, between the limiting ray (from the zone of radius r) and the axis at an axial distance δf from the crossing point of the ray. Then, if f has the same meaning as before,

$$k = \delta f \frac{r}{f} = 2\delta p \frac{f}{r}.$$

Hence W_k, the value of W for a displacement k at the point fixed by δf, is given by

$$W_k = \frac{360^\circ}{\lambda} \cdot \frac{kr}{f} = \frac{360^\circ}{\lambda} \cdot 2\delta p = \frac{720^\circ}{\lambda} \delta p,$$

or $W_k = 2d\omega$, where $d\omega$ is the difference of phase between paraxial and marginal disturbances. The position of the limiting ray for zero aberration is shown in Fig. 2.

In the case of primary spherical aberration, the circle of least confusion lies at three-quarters of the distance from the paraxial to the marginal focus, and is bounded by the intersection of the marginal ray with the ray from the mid-zone of half-radius in the opposite part of the same diameter (see Heath's *Geometrical Optics*, p. 113). The marginal ray is easily drawn, through the marginal focus, by means of the above relation $W_k = 2d\omega$, while the other mid-zone ray is drawn through its axial intersection point (one-quarter of the distance from the paraxial to the marginal focus) at an angle the tangent of which is numerically one-half that of the marginal ray. The radius of the "circle of least confusion" is thus found, the corresponding W value being 360° (see Fig. 3).

In the case of the zonal spherical aberration the circle of least confusion lies at $c_1 = 467^\circ$, and therefore cannot be shown on the diagram (Fig. 5). The W value corresponding to the diffusion disc bounded by the intersection points of the marginal rays with the focus plane in the "$c_1 = 810^\circ$" plane must be, however, equal to $2\,(810^\circ) = 1620^\circ$, whereas the radius of the actual region of maximum concentration is less than is given by $W = 200^\circ$, i.e. less than an eighth of the ray diffusion patch.

Case of no aberration

The tables and curves are given for one side of the focus only, but they would be exactly similar on the other side when plotted in the same way. The note above, regarding the change of scale, should be taken into account. When $c_1 = 90^\circ$, corresponding to a path difference of a quarter of a wave-length, the actual intensity is still 0·8102, but, as the first dark ring of the Airy disc is almost filled with light, the loss of definition in a complex image would probably just be noticeable. Hence a total depth of focus corresponding to a path difference of $\pm \lambda/4$ may be inferred, and the corresponding focal depth in the object space of a lens system may be calculated with the aid of Fermat's theorem. If a is the diameter of the aperture,

d the distance on which the system focusses with accuracy, d_1 and d_2 the near and far distances involving path differences $\pm c$, then it can be shown (12) that

$$\frac{2}{d}=\frac{1}{d_1}+\frac{1}{d_2} \quad \text{and} \quad \frac{1}{d_1}-\frac{1}{d_2}=\frac{16c}{a^2}.$$

Hence, if $$c=\lambda/4, \quad \frac{1}{d_1}-\frac{1}{d_2}=\frac{4\lambda}{a^2}.$$

This result is not applicable to photographic lenses since the initial spherical aberration is comparatively large, and there will also be larger tolerances for differences of phase. The results are more useful for telescopes.

With the aid of the curves of Fig. 2, the general character of the extra-focal appearances will be recognized. At $c_1 = 360°$ (path difference $1{\cdot}0\lambda$) there is zero intensity on the axis. Farther out still, the broad and dark diffraction band is seen to be spreading out from the centre, while light has reappeared on the axis.

Case of primary spherical aberration

The diffraction patterns are seen in Fig. 3 and Table II to be markedly dissimilar on the two sides of the focus. One on one side is a very pronounced set of ring systems, but on the other side appears a gradually weakening patch of light. The intensity at the focus giving least residuals of phase (90° in this case) is still 0·8 of the greatest intensity possible when no aberration is present, but the first dark ring here is not completely devoid of light, and the first bright ring has double

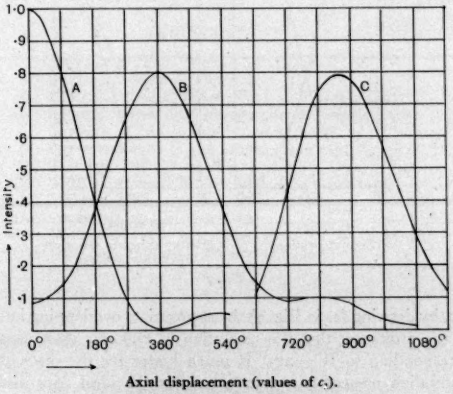

Fig. 6. Distribution of Axial Intensity in three cases:
A. Zero Aberration (Table I).
B. Primary Spherical Aberration (Table II).
C. Zonal Spherical Aberration (Table III).

the intensity of that surrounding the ideal Airy disc. The W value (220°) for the dark ring at this point is practically identical with that of the aberration-free image, a result which has been previously obtained by the writer, using experimental methods. The experiments indicated that this correspondence of size is valid for much higher amounts of primary spherical aberration.

The broad diffraction ring, which begins to develop at $c_1 = 720°$, does not start from the centre, where a central bright spot persists. It will be noticed, from Fig. 6, and it can be shown theoretically, that the axial intensity is symmetrical on each side of the maximum; it only falls away slightly for a change in focus corresponding (as in the case of zero aberration) to a variation of c_1 of $\pm$ 90°. Hence the expression for focal depth will remain approximately the same.

It is instructive to compare the falling off in relative intensity (on proceeding outwards from the axis) of the disc for $c_1 = 450°$ with the ideal Airy disc; the curves suggest a sharper central concentration for the former, since this has its first minimum at $W = 200°$, as compared with $W = 220°$ for the Airy disc. It

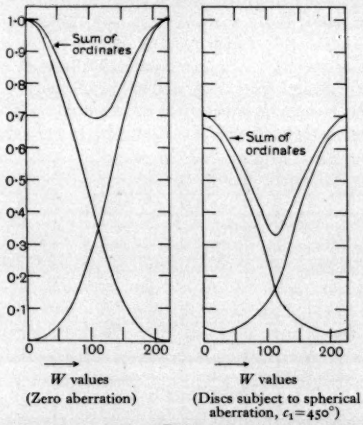

Fig. 7

is easily seen, by drawing (as in Fig. 7) the appropriate overlapping intensity curves and adding the ordinates, that the resolution of two star discs separated by an interval corresponding to $W = 225°$ is much better for the discs obtained with spherical aberration present ($c_1 = 450°$) than for the ideal Airy disc, for in the chosen case the intensity sum at the mid-point sinks to less than half the maximum as compared with 0·7 of the maximum for the Airy disc. Nevertheless, the type of elementary image thus given would undoubtedly prove to be unsatisfactory for

rendering contrast. It is seen that mere resolution, or apparent sharpness of detail, is no criterion of optical perfection, unless accompanied by an image of the maximum possible contrast.

Zonal aberration

It will be noted (see Fig. 6) that the maximum of axial central intensity is not situated at the focus for which the phase residuals are least, i.e. $c_1 = 810°$, but is found at $c_1 = 870°$ approximately; moreover, the distribution of axial intensity is not symmetrical on the two sides of the central maximum as in the previous cases. In comparing the distribution for $c_1 = 855°$ in the zonal case with that for the Airy disc, the radius and darkness of the first interference ring are found to be very similar in each case; but there is much more illumination round the second interference ring in the zonal aberration disc, and this doubtless represents some of the energy lost from the central concentration. There can be no doubt that the resolving power would be practically identical with that characteristic of the aberration-free image, but again there would be a certain loss of contrast. The out-of-focus patterns are characteristically different from either of the other cases. On one side, as at $c_1 = 1170°$, we have a strong ring of illumination which would appear well within the visible disc; on the other side, as at $c_1 = 720°$, there is a very marked dark ring. Effects of this kind are well known to opticians as being characteristic of zonal aberration.

My thanks are due to Professor Conrady, who suggested the work, for his interest and advice.

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DISCUSSION

Mr T. Smith: I should like to congratulate Dr Martin on his paper. It obviously embodies much laborious calculation and ought to be useful to many members of the Society. The list of references omits one paper of importance—that of Struve*. The method of plotting the positions of rays in Fig. 3 is interesting—the intensity scale on the left where the rays cross the axis is rather disturbing, since it is natural

* *Wied. Ann.* 16 (1882) 1008.

to look first to this instead of the scale for c_1, on the opposite side of the diagram. The exact relation satisfied by a ray appears to be

$$W + c_1 (1 + \sec \psi) = 0,$$

where ψ is the inclination of the ray to the axis, so that the deviation from the construction shown will only become significant when the angular aperture is large.

Dr Martin notes that when first and second order aberrations are combined the maximum axial intensity is not found where the variation of phase is least. How this comes about may perhaps be appreciated from diagrams showing the variations in path plotted against the area of the wave front within the zone. In the accompanying figures this area (or $1 - \cos \psi$) is measured vertically. In Fig. *A*

Fig. *A*

Fig. *B*

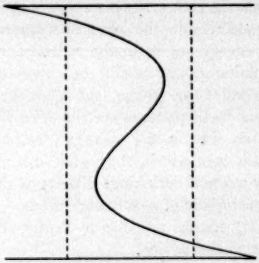

Fig. *C*

the longitudinal aberration is shown measured horizontally and the full vertical line denotes the position of focus for which the range of path variations is least. These minimum path variations are shown in Fig. *B*. If the position of focus is changed this diagram receives a proportional horizontal distortion at such a rate that transference to the position where the rays farthest from the combined paraxial and marginal focus meet the axis will cause the phase to be stationary for the corresponding part of the wave front. If the focus is transferred to the position indicated by a dotted line in Fig. *A*, the variation of phase is shown in Fig. *C*. The greater phase error for the central and marginal zones may obviously be more than compensated by the larger area of the wave near the mean phase, provided the change of focus is not too great. This suggests that the greatest axial intensity will be found rather farther from the paraxial focus than the point where the variation of phase is a minimum.

Prof. Martin: I should like to thank Mr Smith for his remarks, which will emphasise the necessity of referring to the scale for c_1 on the right-hand side of Figs. 2 and 3 when the ray paths are under consideration, and will also remind the reader again of the fact that these diagrams are only a conventional, and not an exact, presentation of the space surrounding the focus.

NOTE ON THE CRITERION FOR THE BEST POSITION OF FOCUS

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(Optics Department, The National Physical Laboratory.)

MS. received, 26th May, 1926. *Read and discussed, 10th June*, 1926.

ABSTRACT. The position in which the amount of energy within the first dark ring of the diffraction image of a point is a maximum is suggested as not unlikely to correspond with the best focus found by visual observation in the presence of moderate amounts of aberration.

The position of best focus is generally assumed by writers on optics to be the plane containing the "least circle of aberration," that is the smallest circle within which all the "rays" of light pass. This concept will appeal to geometricians rather than physicists, for it is obvious that any visual judgment will be greatly influenced by the distribution of the light within an area. For example, when the area is large and the most striking feature is a strong concentration of light towards the centre, the size of the area may easily be disregarded and that position be chosen as the focus where the contrast between the centre and its surroundings is most marked. Geometrical criteria are not likely to receive much consideration at the present time, since it is recognised that the distribution of light must be calculated in accordance with the principles of the wave theory. The best known statement is that of Lord Rayleigh who showed that if the light reaches a particular point with variations of path not exceeding one quarter of a wave length, that point will be accepted as the image about as readily as others where the variations are less. Quite recently a number of investigations have been made where larger variations in path following particular laws have been considered, and the properties particularly noted have been resolving power—usually for point images—and the maximum intensity. The fact that abnormal resolution may be attained by the inclusion of aberrations is of theoretical interest, but it has not yet been shown that optical instruments are improved in performance by retaining in them avoidable aberration. The variation in maximum intensity as the plane is changed can hardly form the criterion adopted in focussing, partly because the eye deals with finite areas and not with points, and partly because it is not a very good instrument for detecting variations of intensity unless the contrast is presented to the eye under well-known conditions which are not satisfied in the process of focussing. It appears that, from whatever point of view it is regarded, both area and intensity are essential factors in the practical problem. The writer is inclined to adopt, at

least as a temporary hypothesis until satisfactory evidence in proof or disproof is available, the view that the position of best focus selected by the eye when moderate aberration is present is very approximately the plane in which the proportion of the total energy lying within the first dark ring is a maximum. The eye is then comparing two fields just distinguishable from one another (since they are separated by the dark ring), and selecting the position where the contrast is as marked as possible. If the aberration is small this position will not differ appreciably from those given by some other criteria, for the diameter of the first ring changes slowly, and the proposal is thus consistent with observation in this field.

Discrimination between one criterion and another in itself may appear a somewhat academic proceeding, but its importance lies in the fact that a knowledge of what the eye accepts will enable us to balance the irremovable aberrations of an optical system to the best advantage, and thus construct better instruments. When the definition is irretrievably bad owing to excessive aberrations it is not of much consequence what position the eye selects, and balancing aberrations represents labour spent in vain. Near the other extreme, when the outstanding aberrations can be so balanced that the variations of path fall within Lord Rayleigh's limit, a balance of the path itself will satisfy all requirements, and has the advantage that it involves the least calculation. Between these two states lies a region of much practical importance, and here it should be both practicable and justifiable to distinguish between the criterion now put forward and those which have been advanced in the past.

DISCUSSION

Prof. L. C. Martin: Mr Smith's suggestion is a very interesting one. Would not the criterion of best focus depend to some extent on the nature of the image? A photographer might sometimes struggle after the very finest definition of detail, irrespective of a little haze; at other times he might be interested in obtaining the maximum contrast.

Mr Smith: I have intentionally referred to the position of focus as it is judged by direct visual examination in order to avoid the additional factors which arise in photography. Different observers seem to choose the same position of focus when the image is examined carefully under suitable magnification, but we do not yet know to what theoretical condition this particular position corresponds. In photography with dry plates the grain usually prevents the recording of fine detail in the optical sense, and I think contrast is much more used as a criterion. A little haze itself tends to obscure fine detail in images produced by photographic lenses.

REVIEWS

Helmholtz's Treatise on Physiological Optics, translated from the third German edition. Edited by Professor James P. C. Southall. Vol. III. *The Perceptions of Vision*. Pp. x + 736; 81 Figs., 6 Plates. (The Optical Society of America, 1925.) Price $7 per volume ($21 for the set of three volumes)†.

The third volume of Prof. Southall's translation of Helmholtz's monumental work *Handbuch der Physiologischen Optik* completes the task so worthily undertaken by the Optical Society of America. This volume deals with the theory of the perceptions of vision and includes extensive notes and appendices by von Kries in addition to the original work of Helmholtz. It contains over 730 pages of which some 220 are the work of von Kries and 17 are devoted to a partial bibliography of the period 1911–1925, the third German edition having been published in 1910. Full subject and author indexes covering the three volumes are included, and corrigenda for volumes I and II complete the volume.

The high standard attained by the translators in the previous volumes has been, if possible, surpassed, the style being clear and free from that cumbersomeness found only too often in translations from the German. Prof. Southall in his introduction refers to the difficulty experienced in translating this volume; this can readily be appreciated by those who have attempted to read German philosophical works in the original, but we can assure the translators of this volume that they have attained a clarity and directness of presentation which could hardly be improved upon. This achievement was made possible by the precision and lucidity of the original author which are rightly acclaimed as marvellous.

The type is clear, but too many printer's errors have been allowed to remain though their number is not great for so large a work. Unfortunately, a large number of the figures, particularly the diagrams, have the appearance of having been printed from worn blocks and therefore leave much to be desired. It is hoped that these imperfections, though they only very slightly dim the lustre of the work, will be removed when the next edition is published.

These three volumes, which are perhaps the greatest contribution made in modern times to the literature of optics, should be accessible to all those interested in the science. The first two volumes have already been given high praise in these pages and no less credit is due to the third volume than to its predecessors.

As pointed out by Prof. Southall, this volume deals rather with psychological than with physiological optics. The controversial area is entered, and it might well be feared that the sands of metaphysics would confuse those wayfarers who dread what they believe to be the heated reflection of sterile minds lest they be led astray by some *fata Morgana* of German philosophical speculation. However, such fears prove to be baseless, and the volume is such delightful reading that it may profitably be studied by the amateur and will certainly interest the dilettante. Chapters are included on the perceptions in general, movements of the eyes*, the monocular field of vision*, the direction of vision*, perception of depth*, binocular double vision*, rivalry between the visual globes*, review of the theories—those marked with an asterisk have been enlarged by the addition of excellent notes by von Kries. The very valuable Appendices I and II by von Kries contain sections on the nature of the idea of space in general, the relations of normal localisation, changes of localisations for anomalous adjustments of the eyes, learning to see and

† This work can be ordered by writing to the Secretary of the Optical Society of America, Rockefeller Hall, Ithaca, N.Y., U.S.A., or may be obtained from the Editor, *The Optician*, 72–78 Fleet St, London, E.C.4.

forgetting, the physiological foundations of judgment and learning, empiricism and nativism, concerning the origin of the laws of ocular movements, historical and critical comments, in Appendix I, while Appendix II is concerned with the theory of binocular instruments.

It is neither necessary nor possible to give a detailed criticism of a work from which spring the majority of the axioms on which are founded the scientific use of the sense of vision to-day. All that can be done is to call attention in general terms to its value to all concerned in optics whether as ophthalmologists or physicists.

If attention has been drawn to the few defects in the presentation of this volume, it is because they are the more apparent by reason of the brilliance of their surroundings from which, however, they cannot effectively detract.

Even to the cursory reader it will become abundantly clear that here is a volume which can be read most profitably whether as a standard text book by those more immediately concerned with its matter, or as a wholesome and, indeed, necessary mental stimulus by those whose study is the more materialistic side of the science. The former, even though they may disagree with certain of the propositions and find a certain amount of obsolescent and obsolete matter, will discover that the greater proportion of the work is of the stuff of which solid foundations are made, and on which they would do well to build. The latter will learn much to assist them to a philosophical appreciation of the processes of which they make daily use in their work, to which many of them may not have devoted more than the superficial attention of a chance discussion.

It is inevitable that metaphysical aspects present themselves since one of the senses is in question, but all students of optics who would attain that breadth of outlook so necessary in any subject if work of lasting quality is to be done, cannot do better than make themselves familiar with this classic work of Helmholtz. Although the temptations must have been such as to attract successfully a lesser man of his race and culture, Helmholtz has never permitted himself to err into the wide and arid wastes of metaphysics in whose sands so many of his countrymen have lost themselves.

His territory is always productive, and Prof. Southall and his collaborators will reap a rich harvest of gratitude from the English-speaking scientific world to whose members they have opened these fertile fields.

G. R. E.

Practical Photo-Micrography. (Second Edition.) By J. E. BARNARD, F.R.S., F.Inst.P., F.R.M.S., and FRANK V. WELCH, F.R.M.S. Pp. xii+316; 86 Figs. and 16 Plates. (London: Edward Arnold & Co., 1925.) Price 18*s*. net.

We heartily welcome the second edition of Mr J. E. Barnard's *Photo-Micrography*, which has been the standard English book on the subject for the last decade and a half. The new edition is written in collaboration with Mr F. V. Welch.

The authors point out that in the meantime there has been no substantial improvement in microscope objectives of the highest quality, though one or two new ones cover a little fresh ground, e.g. Beck's very fine wide-angle 16 mm. apochromat, of which the authors speak in the highest terms, and also the beautiful 8 mm. of the same makers. We should have liked to hear the authors' opinion of Watson's fine 4 mm. apochromat of ·85 N.A., eliminating the extreme outer zone which at present no maker can really correct and which as a matter of experience is not very often of much use, and is sometimes actually harmful. This objective is certainly easier to work with than the more usual ·95 N.A. ones.

An appreciable advance has been made in high power dark ground illuminators, particularly the fine-focussing one of Messrs Beck, of which the authors speak so highly, but they do not make it quite clear that besides the form of 1·27 N.A. which has to be used with special ·5 mm. slips, there is another pattern giving results unequalled by any other

apparatus, on slips of any thickness, with a dark aperture varying from about 1·05 N.A. for a thick slip to 1·2 for a rather thin one.

The authors do not mention Watson's "cassegrain" illuminator. Owing to its extraordinary aperture it has not the sharp focus of some other forms, but it will give a dark ground with a wide aperture immersion lens, using ordinary slides, which cannot be done by any other means.

In photo-micrography the depth of view or "penetration" of the optical combination is often a matter of great importance. We are therefore extremely sorry to notice on p. 23 the statement that penetration "decreases directly as the N.A. of the objective is increased" and "decreases as the square of the focal length"; this latter is clearly intended to mean "varies as the square of the focal length." The consideration of a very simple diagram such as that on p. 22 shows that the penetration varies *directly* as the cotangent of half the angular aperture of the objective in the medium in which the *object* is mounted and *inversely* as the *total* magnification.

The authors' first statement is approximately true where the aperture is small, and is always in the right sense inasmuch as the penetration always diminishes with increase of N.A.

The effect of a denser medium increasing the penetration is rightly insisted on more than once in the book.

The authors' incorrect second statement above is amplified as follows: "a $\frac{1}{2}$ inch objective of 0·5 N.A. has four times the penetration of a $\frac{1}{4}$ inch of the same N.A." Actually, if used with the same eyepiece and tube length, the $\frac{1}{2}$ inch will have *double* the penetration, owing to the lower magnification, while if the same total magnification is obtained by suitable ocular, etc., the penetration will be exactly the same. This mistake is repeated on p. 43.

As this is a point of some importance and interest, we turned up some twenty fairly recent microscopical books, but only four committed themselves to definite statements on the matter. Pringle (3rd edition) and Hind and Randles (1913) both make exactly the same mistake as our authors; Dallinger's *Carpenter* (8th edition) says the penetration varies inversely as the N.A. and inversely as the total magnification; while Spitta (3rd edition) in a chapter written in collaboration with Prof. Conrady gives "approximately" inversely as the N.A. and inversely as the total magnification. The last two books also consider quite accurately the extra depth due to the accommodation of the observer's eye, but in photo-micrography we are obviously not concerned with this.

On p. 52 there is a table of Watson's series of substage condensers giving aplanatic apertures and focal lengths with and without the top lens, and also diameters of the back lens. The figures given are impossible in the case of the "universal" condenser, and extremely improbable for the "parachromatic," when used without the top lens in each case. We find the makers' list gives ·40; 1 in. and ·77 in. respectively, as against ·40; 1·3 in. and ·92 in. in the book for the "universal." The makers state that the figures for the "parachromatic" condenser given in their list are incorrect.

An interesting new feature of this edition is Plate I giving photographs of the rays of light in uranium glass, from various substage illuminators. They form a striking and valuable object lesson.

We gather from the text that Figs. D, E, and F are all Zeiss's aplanatic immersion condenser, not one Leitz and two Zeiss as on the plate.

The first four chapters treat of microscope stands and optical equipment and constitute a useful concise essay on "brass and glass," with little direct reference to photography.

In the next chapter illumination for photography is discussed in detail; Plate III is original and instructive, giving photographs of cold arc light carbons in correct and two wrong positions, accompanied by photographs of the carbon points when the arc is running, viewed from the side and end on in each case.

Chapter VI is perhaps the best in the book, though the choice is not easy to make. The authors here discuss in a masterly manner the pros and cons of vertical and horizontal cameras of every known type, including the recent micro-camera attachments of Leitz and Zeiss.

We are glad that our authors repeatedly insist on the advantage of "geometric" slides and geometric fixed positions; Fig. 38 shows the latter with a flat plate, conical hole, and V groove for the three feet of an arc lamp, but the groove is shown at right angles to its true position, though the text is quite clear. On pp. 121–2 a geometric slide is described and figured, which, instead of bearing on 5 points, bears (if accurately made) on 5 lines! It can, however, be very easily made geometric.

Subsequent chapters consider the manipulation of the microscope in detail and then the combination of microscope, camera, and illuminant, till finally the climax of actual photography is reached.

On p. 141 the alternate use of both eyes is advised, to save fatigue. A word of warning may not be out of place here, as some astronomers and microscopists find that this actually increases the fatigue, probably when there is a considerable difference in the visual acuity of the observer's eyes. In any case the alternate use of the eyes makes it more difficult to keep both eyes open, when using one only.

The authors consider (p. 150) that the thickness of the cover-glass of a mounted object cannot be measured accurately, but with a carefully calibrated microscope fine adjustment we find we can measure the cover-glass at least as accurately as with a simple micrometer, say within 2 or 3 μ—quite sufficient for the purpose.

In discussing cover-glass correction they do not mention the useful method with objects in balsam, etc. of getting correction for any part of the object and doing all further focussing with correction collar or tube-length variation, as the case may be, thus keeping the cover correction accurate however curved or uneven the object may be.

On pp. 219 and 220 the authors give a method of calculating the N.A. of the objective in use, from the diameter of the Ramsden disc, and hence calculating the exposure. This procedure can be simplified since, with the same light source and object, the exposure *varies as the square of the camera extension and inversely as the square of the diameter of the Ramsden disc* for any objective, ocular, tube-length, and substage condenser.

The photographic section is particularly clear and well balanced, striking a happy mean in giving a sufficient choice of methods and processes for every purpose, while avoiding excessive multiplication and confusion.

There are one or two small omissions, e.g. owing to irregularities in glass plates, lantern slides must be made in the camera for very fine detail, not by contact printing. Again, fine detail may be lost in P.O.P. work owing to expansion and contraction, if printing is very slow. The authors mention the danger of staining negatives in prolonged printing, but omit the remedy of varnishing the negative.

On p. 231 an acidified *saturated* solution of mercuric chloride is recommended for bleaching preparatory to intensification, but this is dangerous as a fall in temperature may lead to a deposit of tiny crystals spoiling the negative.

There is a very interesting chapter on "Some Special Processes" including metallography and other vertical and direct illumination work, photography of plate and tube cultures of bacteria, and the authors' specialities of ultra-violet and X-ray photo-micrography.

Last, but far from least, there is the final chapter on "Some Progressive Examples" which consists of 40 beautiful photo-micrographic reproductions on 8 plates, with the most complete particulars of objectives, oculars, condensers, screens, illuminants, exposure, and development, etc. in each case. The reproduction of these plates in half-tone compares wonderfully well with the collotype of the first edition; in fact to the naked eye they actually appear better than the later printed copies of the latter. It is of course well

known that half-tone blocks stand up to the printing press many times better than collotype.

The selection of photo-micrographs has been much improved in this edition; all the best and most interesting (about half) have been retained, while the remainder have been replaced by a rather smaller number fully up to the standard of the best. A very striking thing about these photographs is the extraordinary number of objectives, no less than 20 different ones being used; the fact that more than half are achromatic demonstrates the authors' wonderful mastery of their tools. We are pleased to see that 13 out of these 20 objectives were by English makers, against 6 German and 1 Austrian.

The diatom *Stauroneis phoenicentron* Ehr. in Plate XIV is incorrectly named *Navicula rhomboides*, and the valve is called the frustule.

We only noticed three other misprints which might puzzle a beginner, "ferrous oxalate" for "potassium oxalate" (p. 231, lines 25 and 28) and "concave" for "convex" (p. 164, line 33); we presume "100" (p. 258, line 1) is a misprint for "400" or some similar figure.

The expression "image of the illuminant" as used by the authors appears to refer sometimes to the image of the auxiliary condenser (or intermediate iris) and sometimes to the actual source of light. Similarly "filling the field" (with light) sometimes refers to filling the aperture of the substage condenser and sometimes to filling the field of view.

There is an excellent index with good cross references, but we happened to notice that the Siedentopf photographic eyepiece by Zeiss appears under "Camera," "Siedentopf," and "Zeiss," but not under "Photographic" or "Eyepiece," while the similar "Micca" Micro-Camera by Leitz appears under "Camera" only, not under "Micca," "Micro," or "Leitz."

The bibliography for a work of this standard is very disappointing. It gives four periodicals only, the *J.R.M.S.*, *J.Q.M.C.*, *Q.J.M.S.* and *J. Photo-micro. Soc.*—no continental, American, or photographic journals at all. It gives twenty books, without date, edition, publisher, or country! Of these eighteen are English, one American, and one translation from French; all other foreign books are ignored. One or two of the books included are of only trivial importance, while even such recent English books as Hind and Randles' *Photo-micrography* (Routledge, 1913) and West's *Photo-micrography* (1916) are not given, nor two books mentioned in the authors' own text!

In spite of some imperfections in the book, the authors clearly prove themselves to be past masters in the art of photo-micrography. We note with pleasure that they are to publish a separate book on their recent epoch-making development of ultra-violet photo-micrography.

MAURICE BLOOD.

A Radiometric Investigation of the Germicidal Action of Ultra-Violet Radiation. Scientific Papers of the Bureau of Standards, No. 495. By W. W. COBLENTZ and H. R. FULTON. Pp. 40; 11 Figs. (Washington: Government Printing Office, 1924.) Price 20 cents.

An attempt is here made to estimate the bactericidal action of light by a radiometric method. All workers who have conducted any experiments on the biological effects of light are aware of the need of some reliable method of energy measurement, but it is doubtful whether the authors of this paper have satisfactorily met the difficulty. They describe their principle as being either subtractive or additive. The former is dependent on the use of absorption screens to eliminate different portions of the ultra-violet spectrum; the latter depends on the use of thin mica screens which are said to have an absorption band at about 260 $m\mu$. A small variation in thickness of the mica layer is stated to produce a large variation in transmission in this region, the remainder of the

spectrum being transmitted practically without change. Criticism of these methods is difficult within the limits of a short review but it is permissible to point out the difficulty and uncertainty of using absorption screens for any purpose of accurate calculation in the ultra-violet. Further, most samples of mica will not transmit the region referred to by the authors even in extremely thin laminae. They are quite correct in pointing out that bactericidal action occurs even down to the Schumann region, such action being very intense within the limits of the aluminium lines ranging from about 185 to 200 $m\mu$. The limits of the Schumann region are inaccurately stated to lie between 125 and 170 $m\mu$, whereas 175 $m\mu$ is entirely absorbed by 1 cm. of air. Radiations in the Schumann region are not of necessity of low intensity as stated; their intensity in fact depends on the method of excitation.

Several experimenters have used the ultra-violet spectrum, as produced by a quartz spectrograph, to determine the bactericidal action of various wave-lengths in this region and this method would appear to be more accurate both for determining the active region and for the opportunity it gives of energy measurement. Providing that various energy sources are used, such as the carbon arc for the longer and the condensed spark for the shorter ultra-violet radiations, it would appear to be a more satisfactory method for such purposes than either of those mentioned in this paper.

J. E. B.

Stereoscopic Photography. By ARTHUR W. JUDGE. Pp. xv + 240; 148 Figs. and 19 Plates. (London: Chapman and Hall, Ltd., 1926.) Price 15*s*. net.

This is a book which, from the optical point of view at least, can scarcely be taken seriously. It is abundantly clear that the author is treading ground which is largely unfamiliar to him. This is the more unfortunate, in that he is most insistent upon the necessity of having a correct understanding of the optical principles involved, if the best results in stereoscopic work are to be obtained. It is twice stated (pp. 5 and 7) that the alteration of the configuration of the crystalline lens of the eye enables it to retain the same focal length for all distances of objects. A little later (p. 11), however, the capacity of *altering* its focal length is stated to be "another highly important fact about the eye." Just as with focal length, a similar confusion of ideas appears in connexion with such terms as convergence, parallax, the angular subtense of distant objects, and so on.

A very bad mistake occurs towards the end of the book. There is a method of stereoscopic projection in which the paired images are projected in superposition by beams of light polarised in planes at right angles and are viewed through suitably orientated analysers. In describing this process the author states "...one picture consisting of erect images will be superimposed upon the other picture having its images at right angles, that is, lying upon their sides, owing to the polarised light being at right angles." The same idea is emphasised in the next sentence, and is evidently no slip of the pen. The author clearly thinks that the rotation of a Nicol prism about its axis turns the image round also.

In the light of what has been said it is obvious that no weight can be attached to any theoretical ideas on the subject which the author may present. He refers to papers which have appeared in these *Transactions* under the names of Col. van Albada, Dr French, and the reviewer, but the extent to which he is indebted to them, not only for his matter, but even for its presentation, can only be fully appreciated by those who have these papers under their hand. There is one chapter, however, namely Chapter X, "The Wide-angle Stereogram," which invites more particular attention, in that, with the exception of one or two connecting sentences, it is wholly taken and reprinted *verbatim* from van Albada's paper. It is true that the actual reference is given, and that that writer is also mentioned elsewhere as among those who have been kind enough to help.

This is surely not enough. In such a case the only proper acknowledgment is to make a frank statement at the outset as to the source of the text, and to print it all in inverted commas.

It is not necessary to do more than notice the photographic sections of the book. It is unlikely that anyone will attempt stereoscopic work until he is at least reasonably proficient in ordinary photography. A good deal of space is here occupied with what should be the ordinary stock-in-trade of the photographer, and might well have been omitted, while on the other hand more assistance might have been given upon the essential points of difference which arise as between the two processes. The description of the various pieces of apparatus covers a very extensive range, but at the same time some of it is not closely relevant to the subject proposed. For instance, the stereoscopic photographer is not specially interested in the catalogue of binocular microscopes, which, as the author naively, but quite correctly, remarks, are designed "principally for the purpose of enabling both eyes to be used at the same time." An optical treatment of more of the instruments illustrated would have been interesting, as, for instance, the auto-transposing stereoscope.

The general style of writing, with its crudities of expression, its grammatical lapses, its frequent split infinitives and false "and which's" and even occasional "malapropisms" will jar on the literary purist. The undue emphasis laid upon what are often mere truisms is apt to become irritating, especially in conjunction with frequent repetitions of the same matter, and even a few direct self-contradictions. Reference has already been made to one case in connexion with the focal accommodation of the eye. A worse example is when (p. 22) it is stated that "a mountain some miles away will appear in good relief," and two pages later it is proved that the limit of stereoscopic perception in the normal subject is 670 metres. Three times is an explanation given as to why stereo-prints must be transposed (pp. 80, 104, and 107), following a much earlier statement (p. 39) that "of course" it is necessary to do so in a not very special case. As a final example, we find that four, if not five, times in four pages (pp. 110–113) are the advantages of diffuse side lighting set out in almost identical terms.

The whole treatment, in fact, rather suggests a mass of matter, culled from different sources, and with little attempt at unification, thrown at the reader in more or less undigested lumps, to make what he can of it. The material collected is amply sufficient, and quite suitable to form the basis of a really useful handbook, but if proper use is to be made of it, the present volume will have to be entirely re-written. As it stands, those who are familiar with optical principles cannot fail to realise its errors, while it is doubtful whether others will gather more than a confused and inaccurate impression of stereoscopy. Regretfully one recognises that the gap in photographic literature to which the author refers in his Introduction is not yet filled and cannot be filled except by a writer who possesses considerably greater knowledge of optical science than is shown in this book.

So far as the publishers' work is concerned, one has little criticism to make. The plates giving reproductions of stereo-photographs are admirable. The difficulties of fine-screen process work are by no means small, but something like 300 lines per inch must be used for this class of subject, in order to avoid a very troublesome grain appearing under the magnification of the lenses of the stereoscope. Specimens of Col. van Albada's views, and one or two examples of micro-work by Taverner are very good. All the pairs on Plate I should, however, be transposed. The line stereos in nearly all cases have excessive parallactic displacements, and fusion is only secured, if at all, at the expense of considerable eye-strain. Photographic subjects rarely show displacements of more than a millimetre or so, and in line stereos three millimetres in the standard size picture should not be exceeded. This of course renders direct drawing, without photographic reduction, rather difficult in all but comparatively simple figures.

R. J. TRUMP.

ABSTRACTS OF PATENT SPECIFICATIONS

236626. White, A. E. BIFOCAL LENSES.

The insert and the main lens are both of crown glass of the same dispersion, but the former is of higher refractive index.

236634. Smith, T., and another. TESTING LENSES.

In testing-apparatus of the kind in which the lens under test is rotated about an axis to bring all parts of the image under observation, the arrangements are such as to ensure at all times the correct disposition of the object, real or virtual, the lens, and the image receiver. The auxiliary elements are mechanically constrained so as to move in the direction of their axes, and along the axis of the incident light. A number of arrangements are described, suitable for ordinary lenses, process lenses fitted with reflecting prisms, and telephoto-lenses; and provision may be made for systems which obey the sine or the tangent conditions.

236639. Cubitt, A. S. PROJECTION APPARATUS.

Sharp and diffuse images are projected in succession from slides or films to produce a relief effect. The direct beam may provide the sharp image, and a rotating sector mirror, working in conjunction with a fixed mirror, thereby lengthening the light path, may form the diffuse image. As an alternative to the latter, a mirror which can be deformed to the desired degree by a pressure screw may be employed.

237212*. Ernemann-Werke Akt.-Ges. LENSES.

Numerical data are given for an objective corrected for chromatic and spherical aberration and astigmatism, consisting of a dispersive member between two collective components, of which the front one includes at least two collective elements.

237259*. Chromo Film Ges. OPTICAL SYSTEMS.

In a kinematograph camera arranged to place two images simultaneously in the space usually occupied by one, an optical system consisting of three right-angled prisms, or an equivalent compound single prism, is employed to turn the images through a right angle about the optical axis.

237349. Jarnier, J. H. STEREOSCOPIC APPARATUS.

Stereoscopic pictures for kinematography are taken by a camera having two lenses spaced laterally, but one higher than the other, prisms being employed to place two images, one above the other, in the space usually occupied by one. The paired images are projected simultaneously, and observed through a binocular viewing-apparatus, which includes adjustable prisms to enable the two pictures to be seen superposed.

237529. Ernemann-Werke Akt.-Ges. LENSES.

In a lens consisting of three collective components and a dispersive component, one of the front collective components or the dispersive component is formed as a cemented triplet.

* Specification published before acceptance.

237709. Benton, W. A. PROJECTION LENSES.

Lenses for optical projection consist of a meniscus or plano-convex lens in combination with an over-corrected doublet of smaller diameter, separated by a distance approximately equal to the mean of their focal lengths. The exterior surfaces of the system are both convex.

237861*. Ernemann-Werke Akt.-Ges. LENSES.

A lens as described in Specification No. 237212 is modified by introducing a cemented surface chromatically under-corrected. The resulting zonal spherical aberration is allowed for by using slightly different glasses in the cemented doublets.

238088. Long, E. M. GRINDING LENSES.

In a lens-edging machine in which the lens, held in a rotating holder, is presented to a grinding-wheel having a V-groove for taking off both corner edges of the lens simultaneously, mechanism is provided for producing a forward movement of the grinding-wheel in proportion to the rotation of the lens holder. For this purpose a ratchet feed is applied to a cam, which moves a carriage upon which the wheel is mounted.

238275. Ionides, A. C., and another. OPTICAL APPARATUS.

The invention consists in a reflector system employing, in conjunction with a pair of inclined mirrors, an additional plane or angular reflector fixed or adjustable with respect to the first two. Application is made to gun-sights, range-finders, theodolites, and other instruments for reading angular magnitudes. The inclined mirrors provide two views of a distant object, laterally reversed with respect to each other, and coinciding at the central line, which forms the fiducial mark.

239168*. Wilfred, T. PROJECTION APPARATUS.

A solid object is placed in a beam of light, which throws a shadow upon the screen, and it is also illuminated from the front, a lens being so placed as to throw an image of the illuminated side into the shadow. The object may be moved or rotated under the control of a keyboard, levers, or a perforated sheet, and various lighting effects may be employed.

239508*. Golaz, J. STEREOSCOPIC APPARATUS.

Stereoscopic slides are shown by a projector having two lenses, and fitted with a shutter which unmasks the two views alternately, persistence of vision being relied upon to produce a relief effect. In kinematography, the paired images may be arranged alternately upon the film, and shown in succession.

239677. Kirtane, M. M. PERISCOPES.

The optical system is such that a complete panorama is projected upon a cylindrical screen. The upper prism is in the form of a solid of revolution generated by a right-angled triangle. The bottom prism may be of similar shape, when lenses are fitted in the tube of the periscope to form the necessary image, or its vertical surface may be curved, to form an image directly. The screen may be graduated in degrees, and the fore-and-aft direction of the carrying submarine indicated.

239782. Caughlan, M. W. REFLECTORS.

In a reflector designed for vehicle lamps, the vertical section of the lower half is parabolic, and the horizontal sections on planes through and below the centre are ellipses,

* Specification published before acceptance.

having their outer foci on a vertical line lying in a slit which forms the light-exit aperture of the lamp. The upper half of the reflector may be similar to the lower, or it may be formed as an ellipsoid.

240181. Christiansen, G. O. J. P. VEHICLE LAMPS.

The lamp is egg-shaped, and provided with a downwardly inclined front opening. The axis of the reflector is tilted downwards, as is also the axis of the lamp, but at a smaller angle.

240393. Tumey, C. H., and others. VEHICLE LAMPS.

The lower half of the front lens, which is of hemispherical shape, is left clear, while the upper half is first silvered and then coated with an opaque material. The light which would have been directed upwardly is thereby thrown down to the front and sides of the vehicle.

240426*. Zeiss, C. OPTICAL APPARATUS.

The relative directions of two axes lying in one plane are tested by an apparatus in which are seen simultaneously a sighting mark in the eyepiece of a telescope and the image of a sighting mark in a collimator. Particular forms of apparatus are described for testing the axial directions of various engine parts. The telescope is applied to one of the parts and the collimator to the other, the arrangements being made to suit the test required. Observation may be either direct or by the use of auxiliary systems.

240569. Dorey, W. A. LAMPS.

Selective distribution of light is obtained by the provision of glass panels, of which one surface is shaped to form concentric prismatic corrugations to control the distribution of light in the longitudinal direction, and another surface is provided with parallel corrugations to diffuse and distribute the light laterally.

241131. Golby, F. W. REFLECTORS.

The horizontal section of the reflector of a vehicle lamp consists of two part-parabolas with their axes inclined to each other. The vertical section is a true parabola, or a double parabola of which the upper half has its axis sloping downwards.

241267. McCombie, T. G. F. G. SEXTANTS.

The platforms carrying the working parts are arranged at the corners of an equilateral triangle. The index member is marked with a grooved central line, so that the image of an object in transit is doubled. One half of the horizon mirror is fixed, the other half being attached to a counterweight device. The telescope is also weighted, so as to lie horizontally.

241312. Reid, R. C. GRINDING LENSES.

The lens is rotated beneath a rocking and rotating tool, which is preferably tubular and of smaller diameter than the lens. The tool is universally mounted upon the driving shaft, which is itself carried in a bearing adjustable upon the tool, so that the radius of curvature of the path may be varied.

R. J. T.

* Specification published before acceptance.